Unitarity analysis of a non-Abelian gauge invariant action with a mass

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Abstract

In previous work done by us and coworkers, we have been able to construct a local, non-Abelian gauge invariant action with a mass parameter, based on the nonlocal gauge invariant mass dimension two operator $F_{\mu\nu}(D^2)^{-1}F_{\mu\nu}$. The renormalizability of the resulting action was proven to all orders of perturbation theory, in the class of linear covariant gauges. We also discussed the perturbative equivalence of the model with ordinary massless Yang-Mills gauge theories when the mass is identically zero. Furthermore, we pointed out the existence of a BRST symmetry with corresponding nilpotent charge. In this paper, we study the issue of unitarity of this massive gauge model. Firstly, we provide a short review how to discuss the unitarity making use of the BRST charge. Afterwards we make a detailed study of the most general version of our action, and we come to the conclusion that the model is not unitary, as we are unable to remove all the negative norm states from the physical spectrum in a consistent way.

1 Introduction

In the two previous papers [1, 2], the following action was constructed

$$
\begin{aligned}
S_{phys} &= S_{cl} + S_{gf} \;, \qquad (1.1)\\
S_{cl} &= \int d^4x \left[-\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{im}{4} (B - \overline{B})^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{4} \left(\overline{B}^a_{\mu\nu} D^{ab}_\sigma D^{bc}_\sigma B^c_{\mu\nu} - \overline{G}^a_{\mu\nu} D^{ab}_\sigma D^{bc}_\sigma G^c_{\mu\nu} \right) \right.\\
&- \frac{3}{8} m^2 \lambda_1 \left(\overline{B}^a_{\mu\nu} B^a_{\mu\nu} - \overline{G}^a_{\mu\nu} G^a_{\mu\nu} \right) + m^2 \frac{\lambda_3}{32} \left(\overline{B}^a_{\mu\nu} - B^a_{\mu\nu} \right)^2 \\
&+ \left. \frac{\lambda^{abcd}}{16} \left(\overline{B}^a_{\mu\nu} B^b_{\mu\nu} - \overline{G}^a_{\mu\nu} G^b_{\mu\nu} \right) \left(\overline{B}^c_{\rho\sigma} B^d_{\rho\sigma} - \overline{G}^c_{\rho\sigma} G^d_{\rho\sigma} \right) \right] \;, \qquad (1.2)\\
S_{gf} &= \int d^4x \left(\frac{\xi}{2} b^a b^a + b^a \partial_\mu A^a_\mu + \overline{c}^a \partial_\mu D^{ab}_\mu c^b \right) \;. \qquad (1.3)
\end{aligned}
$$

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The bosonic fields $B^a_{\mu\nu}$, its conjugate $\overline{B}^a_{\mu\nu}$ and the fermionic (ghost) fields $G^a_{\mu\nu}$ and $\overline{G}^a_{\mu\nu}$ are antisymmetric in their Lorentz indices and belong to the adjoint representation. λ^{abcd} is a gauge invariant quartic tensor coupling, subject to a generalized Jacobi identity [3]

$$f^{man}\lambda^{mbcd} + f^{mbn}\lambda^{amcd} + f^{mcn}\lambda^{abmd} + f^{mdn}\lambda^{abcm} = 0\,, \tag{1.4}$$

and to the symmetry constraints

$$\begin{aligned} \lambda^{abcd} &= \lambda^{cdab}\,, \\ \lambda^{abcd} &= \lambda^{bacd}\,, \end{aligned} \tag{1.5}$$

while λ_1 and λ_3 are mass couplings[1].

To avoid confusion, let us mention here that we shall work in Minkowski space throughout this paper, since we plan to come to the canonical quantization. In [1, 2], the action was treated in Euclidean space.

The classical part of the action, S_{cl}, enjoys a non-Abelian gauge invariance generated by

$$\begin{aligned} \delta A^a_\mu &= -D^{ab}_\mu \omega^b\,, \\ \delta B^a_{\mu\nu} &= g f^{abc}\omega^b B^c_{\mu\nu}\,, \\ \delta \overline{B}^a_{\mu\nu} &= g f^{abc}\omega^b \overline{B}^c_{\mu\nu}\,, \\ \delta G^a_{\mu\nu} &= g f^{abc}\omega^b G^c_{\mu\nu}\,, \\ \delta \overline{G}^a_{\mu\nu} &= g f^{abc}\omega^b \overline{G}^c_{\mu\nu}\,, \end{aligned} \tag{1.6}$$

with ω^a parametrizing an arbitrary infinitesimal $SU(N)$ gauge transformation.

Quite obviously, the gauge model (1.1) did not come out of thin air. Our original motivation was based on the quest for a dynamical mass generation mechanism in gauge theories. We do not plan to give a complete overview of this issue, but let us mention that this has been a research topic since long, see e.g. [4] for a seminal work on this.

More recently, work appeared in which a dynamical gluon mass was introduced phenomenologically based on the QCD sum rules [5]. Such a mass can account for $\frac{1}{Q^2}$ power corrections in certain physical correlators [5, 6, 7]. A natural question arising is where this mass scale would originate from? The authors of [6, 7] invoked the condensation of the operator

$$A^2_{\min} = (VT)^{-1} \min_{U\in SU(N)} \int d^4x \left(A^U_\mu\right)^2\,, \tag{1.7}$$

since it is gauge invariant due to the minimization along the gauge orbit[2]. As it is well known, a *local* gauge invariant dimension two operator does not exist in Yang-Mills gauge theories. The nonlocality of (1.7) is best seen when it is expressed as a series in Euclidean space [10]

$$A^2_{\min} = \frac{1}{2VT}\int d^4x \left[A^a_\mu\left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)A^a_\nu - g f^{abc}\left(\frac{\partial_\nu}{\partial^2}\partial A^a\right)\left(\frac{1}{\partial^2}\partial A^b\right)A^c_\nu\right] + \ldots \tag{1.8}$$

[1] In comparison with [1, 2], we changed the sign of m, λ_3, λ_1 and λ^{abcd} to avoid a number of minus signs.

[2] One should however be aware of the problem of gauge (Gribov) ambiguities [8, 9] for determining the global minimum.

which contains the inverse Laplacian $\frac{1}{\partial^2}$ several times. This is a nonlocal operator, as it can be immediately inferred from its formal expression in d dimensions through

$$\frac{1}{\partial_x^2}f(x) = -\frac{\Gamma\left(\frac{d}{2}\right)}{2\pi^{\frac{d}{2}}(d-2)}\int d^dy\frac{f(y)}{|x-y|^{d-2}}\,. \tag{1.9}$$

All efforts so far were concentrated on the Landau gauge $\partial_\mu A_\mu = 0$. The preference for this particular gauge is obvious since the nonlocal expression (1.8) reduces to an (integrated) local operator, more precisely

$$\partial_\mu A_\mu = 0 \Rightarrow A^2_{\min} = (VT)^{-1}\int d^4x A^2_\mu\,. \tag{1.10}$$

In the case of a local operator like A^2_μ, the Operator Product Expansion (OPE), viz. short distance expansion, becomes applicable, and consequently a measurement of the soft (infrared) part $\langle A^2_\mu\rangle_{\rm OPE}$ becomes possible. Such an approach was followed in e.g. [11] by analyzing the appearance of $\frac{1}{Q^2}$ power corrections in (gauge variant) quantities like the gluon propagator or the strong coupling constant, defined in a particular way, from lattice simulations. Let us mention that already two decades ago attention was paid to $\langle A^2_\mu\rangle_{\rm OPE}$ when the OPE was applied to the propagators [12]. This condensate $\langle A^2_\mu\rangle_{\rm OPE}$ can also be related to an effective gluon mass, see e.g. [13].

A more direct approach to a determination of $\langle A^2_\mu\rangle$ in the Landau gauge was presented in [14, 15]. In [14], a meaningful effective potential for the condensation of the local composite operator A^2_μ was constructed, giving evidence of $\langle A^2_\mu\rangle \neq 0$, and as a consequence a nonvanishing gluon mass of a few hundred MeV was found. The renormalizability of this technique was proven to all orders of perturbation theory in [16].

Effective gluon masses have found application in phenomonological studies like [17, 18, 19]. Also lattice simulations of the gluon propagator revealed the need for massive parameters, when the obtained form factors are fitted by means of functional forms [20, 21, 22, 23]. Other approaches to dynamical gluon masses are e.g. [24, 25]. The estimates of these mass parameters are grosso modo all in the same ballpark, ranging from a few hundred MeV up to $1.2\ GeV$.

It is perhaps important to spend a few words at clearing up a common misconception. The concept of a dynamically generated effective gluon mass does not necessarily entail that we are considering massive gauge bosons that are belonging to the physical spectrum, i.e. that are observable particles. At low energies, perturbative QCD expressed in terms of gluons and quarks completely fails, and the effective degrees of freedom become the hadrons. The phenomena we are interested in, in casu the study of the condensates and ensuing dynamical mass generation, occur in a energy window located in between perturbative QCD and the confined region. Perturbation theory still has its validity there, but it gets corrected by nonperturbative effects like condensates. Due to the lack of an explicit knowledge of the correct physical degrees of freedom (the hadrons), we continue to use the gluons as effective degrees of freedom, although we are already out of the energy regime where these might be considered as asymptotic observables. If we cross from high to low energies, the originally massless and physical gluons will not stante pede become confined at the confinement scale, but rather they will behave as a kind of massive quasi particles before getting confined, and this happens at scales that are phenomenologically relevant. This also means that unitarity

Gauge	Operator
linear covariant	$\frac{1}{2}A^a_\mu A^a_\mu$
Curci-Ferrari	$\frac{1}{2}A^a_\mu A^a_\mu + \alpha\overline{c}^a c^a$
maximal Abelian	$\frac{1}{2}A^\beta_\mu A^\beta_\mu + \alpha\overline{c}^\beta c^\beta$
nonlinear class	$\frac{1}{2}A^a_\mu A^a_\mu$

Table 1: Gauges and their renormalizable dimension two operator

in terms of the gluons is not required or even desired. One expects that quasi particles do have a finite lifetime and cannot be observed as asymptotically free particles.

We have already explained the preferred role of the Landau gauge, since in that case a gauge invariant meaning can be assigned to $\langle A^2_\mu\rangle$. Obviously, since we are working in a gauge theory, the condensates influencing physical quantities should be at least gauge invariant. Therefore, it would be nice to have a dimension 2 condensate that could also be treated in other gauges. As the operator A^2_{min} remains nonlocal, it falls beyond the applicability of the OPE. It is also unclear how e.g. renormalizability or an effective potential approach could be established for nonlocal operators. In most covariant gauges, we and collaborators have discussed that other dimension two, renormalizable and local operators exist. We showed that these operators condense and give rise to a dynamical gluon mass, see Table 1 and [26, 27, 28, 29, 30, 31, 32, 33]. Quite recently, it has also been shown that a class of nonlinear covariant gauges enjoys the fact that A^2_μ is multiplicatively renormalizable [34]. In the maximal Abelian gauge, it was found that only the off-diagonal gluons A^β_μ acquire a dynamical mass [31], a fact qualitatively consistent with the lattice results from [22, 23]. Let us also mention that we have been able to make some connection between the various gauges and their dimension two operators by constructing renormalizable interpolating gauges and operators [31, 35]. These can be used to obtain a formal result on the gauge parameter independence of the nonperturbative vacuum energy due to the condensation, which is lower than the perturbative (zero) vacuum energy [30].

A certain disadvantage of the research so far is the explicit gauge dependence of the used operator. We started looking for a gauge invariant dimension two operator, which a fortiori needs to be nonlocal. We would like to develop a consistent (calculational) framework, hence we are almost forced to look for an operator that can be localized by introducing a suitable set of extra fields. From this perspective, A^2_{min} seems to be rather inadequate as it is a infinite series of nonlocal terms. A perhaps more appealing operator is [1]

$$\mathcal{O} = \frac{1}{VT}\int d^4x F^a_{\mu\nu}\left[\left(D^2\right)^{-1}\right]^{ab} F^b_{\mu\nu}\,. \tag{1.11}$$

This operator found already use in the study of a dynamical mass generation in 3-dimensional gauge theories [36]. When we add the operator $\mathcal{O}$ to the Yang-Mills action via

$$S_{YM} - \frac{m^2}{4}\int d^4x F^a_{\mu\nu}\left[\left(D^2\right)^{-1}\right]^{ab} F^b_{\mu\nu}\,, \tag{1.12}$$

we can localize it to

$$S_{YM} + \int d^4x \left[\frac{im}{4}\left(B-\overline{B}\right)^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{4}\left(\overline{B}^a_{\mu\nu} D^{ab}_\sigma D^{bc}_\sigma B^c_{\mu\nu} - \overline{G}^a_{\mu\nu} D^{ab}_\sigma D^{bc}_\sigma G^c_{\mu\nu}\right)\right]\,, \tag{1.13}$$

at the cost of introducing a set of extra fields [1].

The action (1.13) as it stands is however not renormalizable, but we and collaborators have shown that the generalized version (1.1) is renormalizable to all orders in the class of linear covariant gauges, implemented through S_{gf}, in [1, 2]. We have also calculated several renormalization group functions to two loop order, confirming the renormalizability at the practical level. Various consistency checks were at our disposal in order to establish the reliability of these results, e.g. the gauge parameter independence of the anomalous dimension of gauge invariant quantities like g^2, λ^{abcd} or m, or the equality of others, in accordance with the output of the Ward identities in [1]. We refer the reader to [1, 2] for all details concerning the localization procedure or renormalizability analysis, as well as the need for the extra couplings.

Furthermore, we have proven in [2] the perturbative equivalence of the model (1.1) with ordinary Yang-Mills theory in the case that $m \equiv 0$. We notice that this is a nontrivial statement due to the presence of the quartic interaction $\sim \lambda^{abcd}$ in the extra fields. It has an interesting corollary: because we employ a massless renormalization scheme, in casu $\overline{\text{MS}}$, we can set the mass m equal to zero to determine the renormalization group functions of e.g. the coupling constant g^2, the gauge parameter ξ or original Yang-Mills fields. Since both theories are perturbatively equivalent for $m \equiv 0$, the already mentioned renormalization group functions must be identical. This has indeed been confirmed by the explicit results of [1, 2]. In particular, our model is thus asymptotically free at high energies, with or without a mass. At lower energies, nonperturbative effects can set in, completely analogous to the Yang-Mills case.

Summarizing, we have thus found a classically gauge invariant action, which at the quantum level can be renormalized to all orders in at least the class of linear covariant gauges, and as a bonus it is perturbatively equivalent with ordinary Yang-Mills gauge theories for vanishing mass. We can now ask ourselves two questions:

1. If we treat the mass m as a given classical input, can we consider our model as a candidate for a gauge theory with massive excitations? Therefore, we should prove that the theory is unitary, containing massive particles in a suitably defined asymptotic physical subspace. The particles correspond to the elementary excitations of the original fields. As it is well known, proving the unitarity of gauge theories is not a trivial job. A well known proof in the case of Yang-Mills theories based on the BRST symmetry [37, 38], is given in [39].

2. If we do not want to treat our model as one with a given classical mass m, can we dynamically generate it in a selfconsistent way in this case? Said otherwise, can we develop a method to find a reasonable gap equation for this mass? At high energies, the model is massless and the same as Yang-Mills theory, but it might develop a dynamical mass scale at lower energies, without spoiling the gauge invariance. We cannot add mass terms to the Yang-Mills action without spoiling the gauge invariance or renormalizability, but we can add mass terms to our model. Just as for Yang-Mills theories, we expect our model to be confining at lower energies. As we have already mentioned, the original fields can develop a behaviour different from the one expected from perturbation theory in the energy regime in between confinement and the perturbatively accessible high energy region. For example, a gauge invariant mass parameter could be dynamically generated, thereby modifying the propagators in a nonperturbative fash-

ion, and this without the need that these describe asymptotically observable physical particles.

In this paper, we shall provide an answer to the first question by studying the massive gauge model (1.1). More precisely, we shall quantize the model canonically to have a clear particle interpretation of the quantum fields, and we shall find out whether it is possible to define a physical subspace of states endowed with a positive norm. Naively, one might expect the model to be unitary, because the action (1.1) enjoys a BRST symmetry, generated by the nilpotent operator s,

$$
\begin{aligned}
sA_\mu^a &= -D_\mu^{ab}c^b \,, \\
sc^a &= \frac{g}{2}f^{abc}c^a c^b \,, \\
sB_{\mu\nu}^a &= gf^{abc}c^b B_{\mu\nu}^c \,, \\
s\overline{B}_{\mu\nu}^a &= gf^{abc}c^b \overline{B}_{\mu\nu}^c \,, \\
sG_{\mu\nu}^a &= gf^{abc}c^b G_{\mu\nu}^c \,, \\
s\overline{G}_{\mu\nu}^a &= gf^{abc}c^b \overline{G}_{\mu\nu}^c \,, \\
s\overline{c}^a &= b^a \,, \\
sb^a &= 0 \,, \\
s^2 &= 0 \,.
\end{aligned}
\tag{1.14}
$$

It should not come as a surprise that we shall heavily rely on this BRST symmetry to discuss the unitarity of the model. The paper is organized as follows. In section 2, we review how a sensible physical subspace can be defined by using the *free* BRST charge [40, 41]. As a warming up exercise, we apply the results of section 2 to the well known canonical quantization of Yang-Mills gauge theories in section 3, before turning to the explicit quantization of the gauge model (1.1) in section 4. In section 5, we discuss the presence of some extra symmetries which allow to reduce the physical subspace further. Section 6 is devoted to the (free) classical equations of motion and the Fourier decomposition of the solutions. We shall encounter the problem of "multipoles", since the free equations of motion couple different fields to each other. This gives rise to higher derivative decoupled equations of motion. We also pay attention to the BRST charge and Hamiltonian. In section 7, we discuss how to derive the commutation relations between the creation and annihilation operators, without using the brackets between the fields and their conjugate momenta, which we want to avoid, since not all the Fourier components of the fields and momenta are independent. Once this is done, we come to the conclusion that the massive gauge model (1.1) is not unitary, as we end up with negative norm modes in the physical subspace. We are unaware of any step to further reduce this subspace *in a consistent way*[3] to remove these unwanted modes. We end with some conclusions in section 8.

[3]That means compatible with the interactions of the model.

2 A constructive approach to the question of unitarity in gauge theories

In this section, we shall review how we can construct the action S for an interacting gauge theory, if we have a free theory S_0 at our disposal, together with a nilpotent symmetry generator s_0, so that $s_0 S_0 = 0$. The content of this section is mainly based on [40, 41], although here and there we adapted the proofs. We shall only be concerned with the non-reducible case in this paper.

Let us thus start from the free action S_0. This action contains a set of fields, appearing quadratically. It is given that S_0 enjoys a BRST symmetry s_0, with corresponding nilpotent charge $\mathcal{Q}_0$. As a standard example, we can consider the free part of a gauge theory in a particular gauge, with its corresponding gauge fixing part. For example, in the linear covariant gauge we have

$$S_0 = \int d^4x \left[-\frac{1}{4} \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a \right)^2 + b^a \partial_\mu A^{\mu a} + \overline{c}^a \partial^2 c^a + \frac{\xi}{2} b^a b^a \right] . \tag{2.1}$$

The free BRST symmetry is generated by

$$\begin{aligned} s_0 A_\mu^a &= -\partial_\mu c^a \,, \\ s_0 c^a &= 0 \,, \\ s_0 \overline{c}^a &= b^a \,, \\ s_0 b^a &= 0 \,, \\ s_0^2 &= 0 \,. \end{aligned} \tag{2.2}$$

We mention that also the free "ghost part" has to be included in S_0. We may define the ghost charge $\mathcal{Q}_{gh}$. In [40, 41], the ghost charge is not used. We may use it anyhow in the definition of the physical subspace. However, this requirement is a bit redundant. A BRST cohomological analysis (see later) will eventually learn that a physical state counts neither ghosts nor anti-ghosts in the case of Yang-Mills gauge theories.

Unitarity means that we start from a physical state space $\mathcal{H}_{\text{phys}}$, which is a subspace of the total Hilbert state space $\mathcal{H}$. $\mathcal{H}_{\text{phys}}$ should of course be endowed with a positive norm in order to have a sensible probabilistic interpretation of the quantum theory. If we let the states of $\mathcal{H}_{\text{phys}}$ interact, we must end up again in the (same) subspace $\mathcal{H}_{\text{phys}}$. Nonphysical states, which can have negative norms, may contribute to the $\mathcal{S}$-matrix in internal processes, but they cannot appear in the observable sector (the "out"-space), unless perhaps in zero norm combinations.

Consequently, two questions need to be answered:

1. How do we define the physical subspace $\mathcal{H}_{\text{phys}}$?
2. Do the states in the physical subspace $\mathcal{H}_{\text{phys}}$ possess a positive norm?

Let us first explain how we define our physical subspace, starting from the free action. A state $|\psi_p\rangle$ is called physical if

$$|\psi_p\rangle \in \mathcal{H}_{\text{phys}} \Leftrightarrow \mathcal{Q}_0 |\psi_p\rangle = 0 \,, \quad |\psi_p\rangle \neq |\ldots\rangle + \mathcal{Q}_0 |\ldots\rangle \,. \tag{2.3}$$

Physical states are thus defined from the "free" BRST charge $\mathcal{Q}_0$. Since $\mathcal{Q}_0$ is supposed to be nilpotent, states of the form $\mathcal{Q}_0|\ldots\rangle$ are trivially annihilated by $\mathcal{Q}_0$. We notice that these have zero norm[4]. We can identify them with the trivial state, more mathematically speaking this amounts to consider the $\mathcal{Q}_0$ cohomology. In the usual terminology, we define $\mathcal{Q}_0$-closed and $\mathcal{Q}_0$-exact states by

$$|\psi_p\rangle \text{ is } \mathcal{Q}_0\text{-closed} \quad \Leftrightarrow \quad \mathcal{Q}_0|\psi_p\rangle = 0 \Leftrightarrow |\psi_p\rangle \in \mathrm{Ker}\mathcal{Q}_0\,, \tag{2.4}$$

$$|\psi_p\rangle \text{ is } \mathcal{Q}_0\text{-exact} \quad \Leftrightarrow \quad |\psi_p\rangle = \mathcal{Q}_0|\phi\rangle \Leftrightarrow |\psi_p\rangle \in \mathrm{Im}\mathcal{Q}_0\,. \tag{2.5}$$

Since $\mathcal{Q}_0^2 = 0$, every exact state is trivially closed, meaning that $\mathrm{Im}\mathcal{Q}_0 \subset \mathrm{Ker}\mathcal{Q}_0$. Hence, we can reexpress the condition (2.3) as

$$|\psi_p\rangle \in \mathcal{H}_{\text{phys}} \quad \Leftrightarrow \quad |\psi_p\rangle \in \mathrm{cohom}\mathcal{Q}_0 \equiv \frac{\mathrm{Ker}\mathcal{Q}_0}{\mathrm{Im}\mathcal{Q}_0}\,. \tag{2.6}$$

For the moment, we leave open the (key) question whether these states $|\psi_p\rangle$ have a positive norm.

The next problem is whether we can construct an action S compatible with unitarity? Starting from the free action S_0, we can complement it order by order with terms in the coupling constant(s), so that

$$S = S_0 + S_1 + S_2 + \ldots\,. \tag{2.7}$$

The question becomes how to determine the interaction terms S_1, S_2, ..., such that S describes a unitary model? More precisely, having defined a physical subspace by means of (2.6), we would like to construct the action S such that the subspace defined by (2.6) is maintained under time evolution. In the operator language, we must therefore require that the time evolution operator $\mathcal{S}$, given by

$$\mathcal{S} = \mathcal{T}\left[e^{-i\int_{-\infty}^{+\infty} H_{int}(t)dt}\right]\,, \tag{2.8}$$

with $\mathcal{T}$ the usual time-ordering operation, commutes with the operator $\mathcal{Q}_0$. Then clearly the $\mathcal{S}$-matrix will be unitary, as states evolved w.r.t. $\mathcal{S}$ will asymptotically again belong to the (same) physical subspace $\mathcal{H}_{\text{phys}}$.

In order to solve the previous requirement, we prefer to work in the path integral language rather than in the operator language. Let us thus rephrase the previous requirement in the path integral language. From the LSZ reduction formulae, see [42] and [43] for the original paper, we know that the $\mathcal{S}$-matrix elements[5] are determined by the (connected) amputated n-point Green functions, put on-shell. In a rough notation, we can write

$$\langle \vec{k}'_1, \ldots, \vec{k}'_m|\mathcal{S}|\vec{k}_1, \ldots, \vec{k}_n\rangle \sim \langle\Theta|\mathcal{T}\left[\phi_m\phi_n\right]|\Theta\rangle\,, \tag{2.9}$$

where $\sim$ symbolizes all the necessary prefactors, putting it on-shell, amputating and Fourier transforming to momentum space. ϕ_m and ϕ_n are certain functionals of the fields, leading to a $(m+n)$-point function.

Starting from a generic physical state $|\vec{k}_1, \ldots, \vec{k}_n\rangle$ with the property

$$\mathcal{Q}_0|\vec{k}_1, \ldots, \vec{k}_n\rangle = 0\,, \tag{2.10}$$

[4] The BRST charge can be chosen to be Hermitian.

[5] We can restrict ourselves to the connected $\mathcal{S}$-matrix elements.

we are wondering which condition will assure that

$$\langle \vec{k}_1', \ldots, \vec{k}_m' | \mathcal{Q}_0 \mathcal{S} | \vec{k}_1, \ldots, \vec{k}_n \rangle \stackrel{?}{=} \langle \vec{k}_1', \ldots, \vec{k}_m' | \mathcal{S} \mathcal{Q}_0 | \vec{k}_1, \ldots, \vec{k}_n \rangle = 0 \,, \tag{2.11}$$

where the last equality follows from (2.10). As it is well known, we can express the $\mathcal{T}$-ordered product with the path integral, so that we find[6]

$$\langle \vec{k}_1', \ldots, \vec{k}_m' | \mathcal{Q}_0 \mathcal{S} | \vec{k}_1, \ldots, \vec{k}_n \rangle \sim \langle \Theta | \mathcal{T} \left[\left(\mathcal{Q}_0 \phi_m \right) \phi_n \right] | \Theta \rangle = \int dX \left(\mathcal{B}_0 \phi_m \right) \phi_n e^{iS} \,, \tag{2.12}$$

where X represents all the fields. Now, we can write

$$\int dX \left(\mathcal{B}_0 \phi_m \right) \phi_n e^{iS} \quad = \quad \int dX \mathcal{B}_0 \left(\phi_m \phi_n \right) e^{iS} \,, \tag{2.13}$$

since $\mathcal{B}_0 \phi_n = 0$ by virtue of (2.10).

Consider the path integral

$$\int dX \Phi e^{iS} \,, \tag{2.14}$$

for an arbitrary functional Φ of the fields. We perform the transformation of the path integral variables

$$X = X' + \epsilon \mathcal{B}_0 X' \,, \qquad\qquad \epsilon \text{ infinitesimal} \,. \tag{2.15}$$

As $\mathcal{B}_0$ induces a linear transformation, there is no associated Jacobian, and we find

$$\int dX \Phi e^{iS} = \int dX' \Phi' e^{iS'} + \epsilon \int dX' \mathcal{B}_0 \left(\Phi' e^{iS'} \right) \,. \tag{2.16}$$

Dropping the prime again, we find

$$\int dX \mathcal{B}_0 \left(\Phi e^{iS} \right) = 0 \,. \tag{2.17}$$

Taking a look at (2.17), we can be certain that (2.13) holds when we impose

$$\mathcal{B}_0 e^{iS} = 0 \,. \tag{2.18}$$

In order to proceed, we notice that it is in principle sufficient that (2.18) is fulfilled on-shell as the $\mathcal{S}$-matrix is of course considered on-shell. At the level of the action however, we must require that it holds off-shell. Let us introduce a (very) condensed notation for the action of $\mathcal{B}_0$

$$\mathcal{B}_0 = \Delta \phi_0 \frac{\partial}{\partial \phi} \,. \tag{2.19}$$

Implementing (2.18) at lowest order and making it valid off-shell means that

$$\mathcal{B}_0 S_1 \quad = \quad 0 \underbrace{\Rightarrow}_{\text{off-shell}} \Delta \phi_0 \frac{\partial}{\partial \phi} S_1 = -\Delta \phi_1 \frac{\partial S_0}{\partial \phi} \,. \tag{2.20}$$

[6] We shall use the notation $\mathcal{Q}_0$ for the charge, eventually written in terms of creation/annihilation operators, while $\mathcal{B}_0$ represents the functional analog of $\mathcal{Q}_0$.

We already see here that $\mathcal{B}_0$ will get adapted, more precisely we can introduce the modified operator $\mathcal{B}$ by

$$\mathcal{B}_0 \equiv \Delta\phi_0 \frac{\partial}{\partial\phi} \to \mathcal{B} = \mathcal{B}_0 + \Delta\phi_1 \frac{\partial}{\partial\phi}\,, \tag{2.21}$$

so that

$$\mathcal{B}(S_0 + S_1) = 0\,. \tag{2.22}$$

We remind here that all $\mathcal{Q}$'s ($\mathcal{B}$'s) are Grassmann operators.

Since $\mathcal{B}_0$ is nilpotent, we can act with it on (2.20) to find that

$$\begin{aligned} 0 &= \mathcal{B}_0^2 S_1 = \mathcal{B}_0 \left(-\Delta\phi_1 \frac{\partial}{\partial\phi} S_0 \right) \\ &= -\Delta\phi_0 \frac{\partial}{\partial\phi}(\Delta\phi_1) \frac{\partial}{\partial\phi} S_0 + \Delta\phi_1 \Delta\phi_0 \frac{\partial^2}{\partial\phi^2} S_0\,. \end{aligned} \tag{2.23}$$

Acting with $\frac{\partial}{\partial\phi}$ on

$$0 = \mathcal{B}_0 S_0 = \Delta\phi_0 \frac{\partial}{\partial\phi} S_0\,, \tag{2.24}$$

yields

$$\frac{\partial}{\partial\phi}(\Delta\phi_0) \frac{\partial}{\partial\phi} S_0 + \Delta\phi_0 \frac{\partial^2}{\partial\phi^2} S_0 = 0\,. \tag{2.25}$$

Combination of (2.23) and (2.25) learns

$$\Delta\phi_0 \frac{\partial}{\partial\phi}(\Delta\phi_1) \frac{\partial}{\partial\phi} S_0 + \Delta\phi_1 \frac{\partial}{\partial\phi}(\Delta\phi_0) \frac{\partial}{\partial\phi} S_0 = 0\,, \tag{2.26}$$

from which we infer that

$$\Delta\phi_0 \frac{\partial}{\partial\phi}(\Delta\phi_1) \frac{\partial}{\partial\phi} + \Delta\phi_1 \frac{\partial}{\partial\phi}(\Delta\phi_0) \frac{\partial}{\partial\phi} = 0\,. \tag{2.27}$$

The identity (2.27) expresses nothing more than the nilpotency of $Q = Q_0 + Q_1$, given by (2.21) since, at lowest order

$$\begin{aligned} \mathcal{B}^2 &= (\Delta\phi_0 \frac{\partial}{\partial\phi} + \Delta\phi_1 \frac{\partial}{\partial\phi})(\Delta\phi_0 \frac{\partial}{\partial\phi} + \Delta\phi_1 \frac{\partial}{\partial\phi}) \\ &= \Delta\phi_1 \frac{\partial}{\partial\phi}(\Delta\phi_0) \frac{\partial}{\partial\phi} + \Delta\phi_1 \Delta\phi_0 \frac{\partial^2}{\partial\phi^2} + \Delta\phi_0 \Delta\phi_1 \frac{\partial^2}{\partial\phi^2} + \Delta\phi_0 \frac{\partial}{\partial\phi}(\Delta\phi_1) \frac{\partial}{\partial\phi} + HOT \\ &= \Delta\phi_1 \frac{\partial}{\partial\phi}(\Delta\phi_0) \frac{\partial}{\partial\phi} + \Delta\phi_0 \frac{\partial}{\partial\phi}(\Delta\phi_1) \frac{\partial}{\partial\phi} = (2.27) = 0\,, \end{aligned} \tag{2.28}$$

where we used for example the nilpotency of $\mathcal{B}_0$. We dropped the last term as it is of higher order.

We notice that the potential solution of (2.20) is apparently restrained by the condition that it is invariant under a nilpotent operator Q ($\mathcal{B}$), which reduces to Q_0 ($\mathcal{B}_0$) in the free limit.

This construction can be continued at higher order. One proves that the action at order n,

$$S = S_0 + S_1 + \ldots + S_n\,, \tag{2.29}$$

is the solution of

$$\Delta\phi_0 \frac{\partial}{\partial\phi} S_n + \Delta\phi_1 \frac{\partial}{\partial\phi} S_{n-1} + \ldots + \Delta\phi_n \frac{\partial}{\partial\phi} S_0 = 0\,, \tag{2.30}$$

where consistency demands that the BRST operator,

$$\mathcal{B} = \mathcal{B}_0 + \ldots + \mathcal{B}_n = \Delta\phi_0 \frac{\partial}{\partial\phi} + \ldots + \Delta\phi_n \frac{\partial}{\partial\phi}\,, \tag{2.31}$$

is nilpotent at the considered order n, thus

$$\mathcal{B}^2 = 0 \quad (\text{or } Q^2 = 0)\,. \tag{2.32}$$

By construction, the final action (2.29) shall be invariant under the BRST symmetry generated by (2.31).

To make things a bit more comprehensible, let us work out the procedure at second order. We hence demand that (2.7) is consistent with (2.18), and this extended to the off-shell level, meaning that

$$i\mathcal{B}_0 S_2 - \frac{1}{2}\mathcal{B}_0(S_1 S_1) = -i\widetilde{\Delta\phi_2}\frac{\partial S_0}{\partial\phi}\,. \tag{2.33}$$

The complex unity i in the r.h.s. as well as the $\widetilde{\ }$-notation are merely introduced for later convenience. Using (2.20), we may rewrite (2.33) as

$$i\Delta\phi_0 \frac{\partial S_2}{\partial\phi} + \Delta\phi_1 \frac{\partial S_0}{\partial\phi} S_1 = -i\widetilde{\Delta\phi_2}\frac{\partial S_0}{\partial\phi}\,. \tag{2.34}$$

Next, using Wick's theorem, we can write[7]

$$\Delta\phi_1 \frac{\partial S_0}{\partial\phi} S_1 = i\Delta\phi_1 \frac{\partial S_1}{\partial\phi} + i\widetilde{\Delta\phi_1}\frac{\partial S_0}{\partial\phi}\,, \tag{2.35}$$

since roughly said, $\frac{\partial S_0}{\partial\phi} \sim \mathcal{D}^{-1} \times \phi$, and a "contraction" of this with a ϕ from S_1 will give rise to a $i\mathcal{D}^{-1} \times \mathcal{D}$ with $\mathcal{D}$ a free propagator. All other terms are taken together in $\widetilde{\Delta\phi_1}$.

Upon taking (2.34) and (2.35) into account, we come to the conclusion that

$$\Delta\phi_0 \frac{\partial S_2}{\partial\phi} + \Delta\phi_1 \frac{\partial S_1}{\partial\phi} = -\Delta\phi_2 \frac{\partial S_0}{\partial\phi} \tag{2.36}$$

in order to have the condition (2.33) fulfilled. We defined

$$\Delta\phi_2 = \widetilde{\Delta\phi_2} + \widetilde{\Delta\phi_1}\,. \tag{2.37}$$

Analogously at is was proven in (9) to (14), the nilpotency of $Q_0 + Q_1$ leads to the nilpotency of $Q_0 + Q_1 + Q_2$ as a consistency requirement.

Of course, there is no guarantee that the foregoing "bottom top" construction of the complete action will end at a finite order. Given that it ends at a finite order, it could still be a very cumbersome job to actually get the nilpotent BRST charge and corresponding action. The situation becomes much more appealing when we already have at our disposal a complete

[7] This operation is understood within the path integral.

action, with a nilpotent charge generating a symmetry. If the interaction is switched off by setting all coupling constants equal to zero, we obtain the free action, with a free nilpotent charge. When the above "bottom top" machinery is unleashed, the complete original action and its BRST symmetry generator shall quite evidently be a solution to the iterative procedure. From this viewpoint, we have a "top bottom" approach to unitarity for actions with nilpotent BRST charge, when they are "reduced" to their free counterpart.

3 Unitarity of Yang-Mills gauge theories using the BRST charge

We should still provide an answer to question 2, namely do the states that are annihilated by the free BRST charge have a positive norm? It is well known that this is the case for Yang-Mills gauge theories. For completeness, let us nevertheless repeat the argument. This will allow for a comparison with Yang-Mills theories when we start analyzing our generalized model.

We shall base ourselves on [44] for this particular job[8]. We opt to work in the Feynman gauge for simplicity ($\xi = 1$ in (2.1)). Let us first determine the conjugate momenta of all fields.

$$\begin{aligned} \pi_i^a &= -F_{0i}^a \,, \\ \pi^{0,a} &= b^a \,, \\ \pi_c^a &= \partial_0 \overline{c}^a \,, \\ \pi_{\overline{c}}^a &= -\partial_0 c^a \,, \end{aligned} \tag{3.1}$$

so that quantization requires

$$\begin{aligned} \left[A_i^a(\vec{x},t), \pi_j^b(\vec{y},t)\right] &= i\delta^{ab} g_{ij} \delta^{(3)}(\vec{x}-\vec{y}) \,, \\ \left[A_0^a(\vec{x},t), \pi^{0,b}(\vec{y},t)\right] &= i\delta^{ab} \delta^{(3)}(\vec{x}-\vec{y}) \,, \\ \left\{c^a(\vec{x},t), \pi_c^b(\vec{y},t)\right\} &= i\delta^{ab} \delta^{(3)}(\vec{x}-\vec{y}) \,, \\ \left\{\overline{c}^a(\vec{x},t), \pi_{\overline{c}}^b(\vec{y},t)\right\} &= i\delta^{ab} \delta^{(3)}(\vec{x}-\vec{y}) \,, \\ \text{other (anti-)commutators trivial} \,. & \end{aligned} \tag{3.2}$$

We mention that the classical equations of motion are

$$\begin{aligned} \partial^2 A_\mu^a &= 0 \,, \\ \partial^2 c^a &= \partial^2 \overline{c}^a = 0 \,, \\ b^a &= -\partial^\mu A_\mu^a \,, \end{aligned} \tag{3.3}$$

and that we use the hermiticity assignment

$$c^\dagger = c \,, \qquad \overline{c}^\dagger = -\overline{c} \,. \tag{3.4}$$

[8] We shall however use other conventions than those of [44].

We propose the following Fourier decompositions[9]

$$\begin{aligned}
A_0(x) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left(a_0(k)e^{-ikx}+a_0^\dagger(k)e^{ikx}\right)\,,\\
A_i(x) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\sum_{m=1}^{3}\left(a_m(k)\varepsilon_i^m(\vec{k})e^{-ikx}+a_m^\dagger(k)\varepsilon_i^m(\vec{k})e^{ikx}\right)\,,\\
b(x)\equiv\pi_0(x) &= i\int \frac{d^3k}{(2\pi)^3}\frac{1}{2}\left((a_0(k)-a_3(k))e^{-ikx}-(a_0^\dagger(k)-a_3^\dagger(k))e^{ikx}\right)\,,\\
c(x) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left(\eta(k)e^{-ikx}+\eta^\dagger(k)e^{ikx}\right)\,,\\
\overline{c}(x) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left(\overline{\eta}(k)e^{-ikx}-\overline{\eta}^\dagger(k)e^{ikx}\right)\,.
\end{aligned}\tag{3.5}$$

The polarization vectors $\varepsilon_i^{(m)}(\vec{k})$ form an orthonormal set, $\varepsilon_i^{(m)}(\vec{k})\varepsilon_i^{(n)}(\vec{k})=\delta^{mn}$ with $\varepsilon_i^{(3)}(\vec{k})=\frac{k_i}{|\vec{k}|}$. We shall assume that the particles move along the z-axis, so that $\varepsilon_i^j=\delta_{ij}$.

Implementing (3.2), we must require the following (anti-)commutation rules

$$\begin{aligned}
[a_0(k),a_0(q)] &= -2(2\pi)^3\omega_k\delta^{(3)}(\vec{k}-\vec{q})\,,\\
[a_m(k),a_n(q)] &= 2(2\pi)^3\omega_k\delta^{(3)}(\vec{k}-\vec{q})\delta_{mn}\,,\\
\left\{\overline{\eta}(k),\eta^\dagger(q)\right\} &= -2(2\pi)^3\omega_k\delta^{(3)}(\vec{k}-\vec{q})\,,\\
\left\{\eta(k),\overline{\eta}^\dagger(q)\right\} &= -2(2\pi)^3\omega_k\delta^{(3)}(\vec{k}-\vec{q})\,.
\end{aligned}\tag{3.6}$$

For later use, let us already introduce the operator [44]

$$\mathcal{N}=\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\left(-a_0^\dagger(k)a_0(k)+a_3^\dagger(k)a_3(k)-\eta^\dagger(k)\overline{\eta}(k)-\overline{\eta}^\dagger(k)\eta(k)\right)\,,\tag{3.7}$$

which counts the unphysical modes.

We are now ready to express the BRST charge in terms of the creation/annihilation operators. The BRST Noether current is given by

$$\mathcal{J}_0^\mu = F^{\mu\nu,a}\partial_\nu c^a-b^a\partial_\mu c^a\,,\tag{3.8}$$

which leads to the charge

$$\mathcal{Q}_0 = \int d^3x\left(c^a\partial^0 b^a-b^a\partial^0 c^a\right)\,,\tag{3.9}$$

where use was made of the classical equation of motion

$$\partial_\mu F^{\mu\nu,a}=\partial^\nu b^a\,.\tag{3.10}$$

After substitution of (3.5) in (3.9), the BRST charge is expressed as

$$\mathcal{Q}_0 = \int\frac{d^3k}{(2\pi)^3}\left[\left(a_0^\dagger(k)-a_3^\dagger(k)\right)\eta(k)+\eta^\dagger(k)\left(a_0(k)-a_3(k)\right)\right]\,.\tag{3.11}$$

[9] We suppressed the global color indices.

If we define

$$\mathcal{R} = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\frac{1}{4\omega_k^2}\left[\left(a_0^\dagger(k)+a_3^\dagger(k)\right)\overline{\eta}(k)+\overline{\eta}^\dagger(k)\left(a_0(k)+a_3(k)\right)\right] , \tag{3.12}$$

then a little algebra yields

$$\mathcal{N} = \left\{\mathcal{Q}_0, \mathcal{R}\right\} . \tag{3.13}$$

The fact that the "nonphysical" counting operator $\mathcal{N}$ is BRST exact is a very powerful result [44]. Assume that $|\psi_p\rangle$ is constrained by

$$\mathcal{Q}_0|\psi_p\rangle = 0 , \tag{3.14}$$

and that it contains $n \neq 0$ unphysical modes, i.e.

$$\mathcal{N}|\psi_p\rangle = n|\psi_p\rangle , \tag{3.15}$$

then consequently

$$|\psi_p\rangle = \frac{\mathcal{N}}{n}|\psi_p\rangle = \frac{1}{n}\left(\mathcal{Q}_0\mathcal{R}+\mathcal{R}\mathcal{Q}_0\right)|\psi_p\rangle = \mathcal{Q}_0\left(\frac{1}{n}\mathcal{R}|\psi_p\rangle\right) , \tag{3.16}$$

meaning that a state $|\psi_p\rangle$ annihilated by the BRST charge and containing nonphysical modes is a fortiori BRST exact, and hence it is zero in the physical cohomology. Said otherwise, physical states do not contain unphysical modes. The physical subspace $\mathcal{H}_{\text{phys}}$ of Yang-Mills gauge theories does only contain the 2 transverse polarizations of the gauge field, whereas the scalar and longitudinal polarizations cancel with the ghost degrees of freedom.

In [40], a different proof was presented of the fact that a state annihilated by a BRST charge $\mathcal{Q}_0$ of the form (3.11) containing nonphysical modes, must have zero norm. However, the cohomological approach used in e.g. [44] is somewhat more elegant.

4 Application to the massive gauge model: preliminary remarks

Setting the couplings g and λ^{abcd} equal to zero in (1.1), we are considering the quadratic action

$$\begin{aligned} S_0 &= \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A_\nu^a-\partial_\nu A_\mu^a\right)^2+\frac{im}{4}(B-\overline{B})^a_{\mu\nu}\left(\partial^\mu A^{\nu a}-\partial^\nu A^{\mu a}\right)\right.\\ &+ \frac{1}{4}\left(\overline{B}^a_{\mu\nu}\partial^2 B^{\mu\nu,a}-\overline{G}^a_{\mu\nu}\partial^2 G^{\mu\nu a}\right)-\frac{3}{8}m^2\lambda_1\left(\overline{B}^a_{\mu\nu}B^{\mu\nu a}-\overline{G}^a_{\mu\nu}G^{\mu\nu a}\right)\\ &+ \left. m^2\frac{\lambda_3}{32}\left(\overline{B}^a_{\mu\nu}-B^a_{\mu\nu}\right)^2+b^a\partial_\mu A^{\mu a}+\overline{c}^a\partial^2 c^a+\frac{\xi}{2}b^a b^a\right] \end{aligned} \tag{4.1}$$

This action enjoys the free BRST symmetry generated by

$$\begin{aligned} s_0 A_\mu^a &= -\partial_\mu c^a ,\\ s_0 c^a &= 0 ,\\ s_0 B^a_{\mu\nu} &= s_0\overline{B}^a_{\mu\nu} = s_0 G^a_{\mu\nu} = s_0\overline{G}^a_{\mu\nu} = 0 ,\\ s_0\overline{c}^a &= b^a ,\\ s_0 b^a &= 0 , \end{aligned} \tag{4.2}$$

where clearly

$$s_0^2 = 0\,. \tag{4.3}$$

We can hence apply the results of section 2 to the action (1.1). The only thing left to prove is that there exist a physical subspace with positive norm. This subspace is certainly annihilated by the (free) BRST charge, but nothing prevents us from using other available symmetries to further reduce the physical subspace. In the next section, we shall introduce 2 extra symmetries with nilpotent generator of the complete action (1.1). We first determine the BRST charge in functional form. We shall see that it remains unchanged compared to the Yang-Mills case (3.9). The Noether current corresponding to the BRST transformation (4.2) and action (4.1) is given by

$$\mathcal{J}_0^\mu = F^{\mu\nu,a}\partial_\nu c^a - b^a\partial^\mu c^a - \frac{im}{2}\left(B-\overline{B}\right)^{\mu\nu,a}\partial_\nu c^a\,, \tag{4.4}$$

which leads to the BRST charge

$$\begin{aligned}\mathcal{Q}_0 &= \int d^3x \left(F^{0i}\partial_i c^a - b^a\partial^0 c^a - \frac{im}{2}\left(B-\overline{B}\right)^{0i,a}\partial_i c^a\right) \\ &= \int d^3x\left(c^a\partial^0 b^a - b^a\partial^0 c^a\right)\,,\end{aligned} \tag{4.5}$$

where we invoked the equation of motion

$$\partial_\mu F^{\mu\nu} = \partial^\nu b^a + \frac{im}{2}\partial_\mu\left(B-\overline{B}\right)^{\mu\nu,a}\,. \tag{4.6}$$

For what concerns the Faddeev-Popov ghosts c and $\overline{c}$, it is immediately seen from the action (4.1) that their quantization remains unchanged compared to the Yang-Mills case, given in (3.1), (3.5) and (3.6). Therefore, since (4.5) must be time independent as a conserved charge, we already infer that $\mathcal{Q}_0$ will only act nontrivially on massless excitations. This shall be confirmed later once we have found the excitations belonging to the b^a-field (see section 6).

5 A further reduction of the physical subspace

In the following sections, we shall make use of a cohomological result [45], summarized here.

Doublet theorem Consider a transformation δ with the property that

$$\begin{array}{ll}\delta u_i = v_i\,, & \delta u'_i = v'_i\,,\\ \delta v_i = 0\,, & \delta v'_i = 0\,,\end{array} \tag{5.1}$$

with u_i, v'_i commuting and u'_i, v_i anticommuting quantities. We call (u_i, v_i) and (u'_i, v'_i) δ-doublets.

Then it is a trivial exercise to show that δ is a nilpotent transformation. Moreover, (u_i, v_i) and (u'_i, v'_i) appear trivially in the δ-cohomology. This can be proven [45] by introducing the "counting" operator

$$\mathcal{P} = \int d^4x\left(u_i\frac{\delta}{\delta u_i} + v_i\frac{\delta}{\delta v_i} + u'_i\frac{\delta}{\delta u'_i} + v'_i\frac{\delta}{\delta v'_i}\right)\,, \tag{5.2}$$

and the operator

$$\mathcal{A} = \int d^4x \left(u_i \frac{\delta}{\delta v_i} + u_i' \frac{\delta}{\delta v_i'} \right) , \tag{5.3}$$

such that

$$\mathcal{P} = \{\delta, \mathcal{A}\} \qquad\qquad [\mathcal{P}, \delta] = 0 \,. \tag{5.4}$$

Assuming that $\delta X = 0$, we can expand X in an mutual eigenbasis of the commuting Hermitian operators $\mathcal{P}$ and δ. It is then quite easy to show that

$$X = X_0 + \delta Y \,, \tag{5.5}$$

whereby $\mathcal{P} X_0 = 0$, i.e. the cohomology of δ does not depend on (u_i, v_i) and (u_i', v_i').

In the paper [2], we already noted the equivalence between the action (1.1) for $m \equiv 0$ and conventional Yang-Mills theories, when quantized using the same gauge fixing. The physical content of both theories should be the same, in casu the unitarity should be satisfied. This can be shown in the following way. The action (1.1) enjoys a supersymmetry when $m \equiv 0$, generated by the nilpotent transformation [2]

$$\begin{aligned} &\delta_s B^a_{\mu\nu} = G^a_{\mu\nu} \,, &&\delta_s \overline{G}^a_{\mu\nu} = \overline{B}^a_{\mu\nu} \,, \\ &\delta_s G^a_{\mu\nu} = 0 \,, &&\delta_s \overline{B}^a_{\mu\nu} = 0 \,. \end{aligned} \tag{5.6}$$

So, the **Doublet theorem** applies, and we conclude that the excitations belonging to the extra fields will not belong to the physical subspace.

In the case that $m \neq 0$, the supersymmetry (5.6) is broken due to the terms $\sim (B - \overline{B})$. In a matter of speaking, the symmetry is only broken by terms $\sim (B - \overline{B})$, and not by terms $\sim (B + \overline{B})$. Therefore, we might expect that some trace of the supersymmetry δ_s might survive after all. We shall first explore this possibility. We decompose the fields $G^a_{\mu\nu}$ and $\overline{G}^a_{\mu\nu}$ in their "electric" and "magnetic" part

$$\begin{aligned} &\alpha^{i,a} = \tfrac{1}{2} \varepsilon^{ijk} G^a_{jk} \,, &&\overline{\alpha}^{i,a} = \tfrac{1}{2} \varepsilon^{ijk} \overline{G}^a_{jk} \,, \\ &\beta^{i,a} = G^{oi,a} \,, &&\overline{\beta}^{i,a} = \overline{G}^{oi,a} \,. \end{aligned} \tag{5.7}$$

Consequently, one finds

$$-\frac{1}{4} \overline{G}^a_{\mu\nu} \partial^2 G^{\mu\nu,a} = \frac{1}{2} \overline{\beta}^{i,a} \partial^2 \beta^{i,a} - \frac{1}{2} \overline{\alpha}^{i,a} \partial^2 \alpha^{i,a} \,. \tag{5.8}$$

Since G and $\overline{G}$ always appear in the "product" combination $\overline{G}^a_{\mu\nu} \mathcal{O}^{ab} G^{\mu\nu,b}$, only the structures $\overline{\beta} \mathcal{O} \beta$ and $\overline{\alpha} \mathcal{O} \alpha$ will appear. Having a look at the complete action (1.1), it is clear that also in the interacting theory, these are the only possibly appearing structures.

It is convenient to also decompose the B and $\overline{B}$-fields in their electric and magnetic counterparts

$$\begin{aligned} &\rho^{i,a} = \tfrac{1}{2} \varepsilon^{ijk} B^a_{jk} \,, &&\overline{\rho}^{i,a} = \tfrac{1}{2} \varepsilon^{ijk} \overline{B}^a_{jk} \,, \\ &\sigma^{i,a} = B^{oi,a} \,, &&\overline{\sigma}^{i,a} = \overline{B}^{oi,a} \,. \end{aligned} \tag{5.9}$$

in which case the (quadratic) action becomes

$$
\begin{aligned}
S_0 &= \int d^4x \left[-\frac{1}{4} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu \right)^2 - \frac{im}{2} (\sigma - \overline{\sigma})^{i,a} \left(\partial^0 A^{ia} - \partial^i A^{0a} \right) \right. \\
&+ \frac{im}{2} \varepsilon^{ijk} (\rho - \overline{\rho})^{k,a} \left(\partial^i A^{ja} \right) - \frac{1}{2} \overline{\sigma}^{i,a} \partial^2 \sigma^{i,a} + \frac{1}{2} \overline{\rho}^{i,a} \partial^2 \rho^{i,a} + \frac{1}{2} \overline{\beta}^{i,a} \partial^2 \beta^{i,a} - \frac{1}{2} \overline{\alpha}^{i,a} \partial^2 \alpha^{i,a} \\
&- \frac{3}{4} m^2 \lambda_1 \left(-\overline{\sigma}^{i,a} \sigma^{i,a} + \overline{\rho}^{i,a} \rho^{i,a} + \overline{\beta}^{i,a} \beta^{i,a} - \overline{\alpha}^{i,a} \alpha^{i,a} \right) \\
&+ m^2 \frac{\lambda_3}{8} \left(\overline{\sigma}^{i,a} \sigma^{i,a} - \overline{\rho}^{i,a} \rho^{i,a} \right) + m^2 \frac{\lambda_3}{16} \left(-\overline{\sigma}^{i,a} \overline{\sigma}^{i,a} - \sigma^{i,a} \sigma^{i,a} + \overline{\rho}^{i,a} \overline{\rho}^{i,a} + \rho^{i,a} \rho^{i,a} \right) \\
&+ \left. b^a \partial_\mu A^{\mu a} + \overline{c}^a \partial^2 c^a + \frac{\xi}{2} b^a b^a \right] . \qquad (5.10)
\end{aligned}
$$

For further convenience, we shall exchange $(\sigma, \overline{\sigma})$ and $(\rho, \overline{\rho})$ for their real and imaginary parts via

$$
\begin{aligned}
K^{i,a} &= \tfrac{1}{2}(\rho^{i,a} + \overline{\rho}^{i,a}) , & M^{i,a} &= \tfrac{1}{2}(\sigma^{i,a} + \overline{\sigma}^{i,a}) , \\
L^{i,a} &= \tfrac{1}{2i}(\rho^{i,a} - \overline{\rho}^{i,a}) , & N^{i,a} &= \tfrac{1}{2i}(\sigma^{i,a} - \overline{\sigma}^{i,a}) ,
\end{aligned} \qquad (5.11)
$$

yielding

$$
\begin{aligned}
S_0 &= \int d^4x \left[-\frac{1}{4} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu \right)^2 + m N^{i,a} \left(\partial^0 A^{ia} - \partial^i A^{0a} \right) - m \varepsilon^{ijk} L^{k,a} \left(\partial^i A^{ja} \right) \right. \\
&- \frac{1}{2} M^{i,a} \partial^2 M^{i,a} - \frac{1}{2} N^{i,a} \partial^2 N^{i,a} + \frac{1}{2} K^{i,a} \partial^2 K^{i,a} \\
&+ \frac{1}{2} L^{i,a} \partial^2 L^{i,a} + \frac{1}{2} \overline{\beta}^{i,a} \partial^2 \beta^{i,a} - \frac{1}{2} \overline{\alpha}^{i,a} \partial^2 \alpha^{i,a} \\
&- \frac{3}{4} m^2 \lambda_1 \left(-M^{i,a} M^{i,a} - N^{i,a} N^{i,a} + K^{i,a} K^{i,a} + L^{i,a} L^{i,a} + \overline{\beta}^{i,a} \beta^{i,a} - \overline{\alpha}^{i,a} \alpha^{i,a} \right) \\
&+ \left. m^2 \frac{\lambda_3}{4} \left(N^{i,a} N^{i,a} - L^{i,a} L^{i,a} \right) + b^a \partial_\mu A^{\mu a} + \overline{c}^a \partial^2 c^a + \frac{\xi}{2} b^a b^a \right] . \qquad (5.12)
\end{aligned}
$$

As a final step, we introduce the fields

$$
\chi^{\pm,i,a} = K^{i,a} \pm M^{i,a} , \qquad (5.13)
$$

to write

$$
\begin{aligned}
S_0 &= \int d^4x \left[-\frac{1}{4} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu \right)^2 + m N^{i,a} \left(\partial^0 A^{ia} - \partial^i A^{0a} \right) - m \varepsilon^{ijk} L^{k,a} \left(\partial^i A^{ja} \right) \right. \\
&+ \frac{1}{2} \chi^{+,i,a} \partial^2 \chi^{-,i,a} - \frac{1}{2} N^{i,a} \partial^2 N^{i,a} + \frac{1}{2} L^{i,a} \partial^2 L^{i,a} + \frac{1}{2} \overline{\beta}^{i,a} \partial^2 \beta^{i,a} - \frac{1}{2} \overline{\alpha}^{i,a} \partial^2 \alpha^{i,a} \\
&- \frac{3}{4} m^2 \lambda_1 \left(\chi^{+,i,a} \chi^{-,i,a} - N^{i,a} N^{i,a} + L^{i,a} L^{i,a} + \overline{\beta}^{i,a} \beta^{i,a} - \overline{\alpha}^{i,a} \alpha^{i,a} \right) \\
&+ \left. m^2 \frac{\lambda_3}{4} \left(N^{i,a} N^{i,a} - L^{i,a} L^{i,a} \right) + b^a \partial_\mu A^{\mu a} + \overline{c}^a \partial^2 c^a + \frac{\xi}{2} b^a b^a \right] . \qquad (5.14)
\end{aligned}
$$

We introduce the following transformations

$$
\begin{aligned}
\Delta_\alpha \overline{\alpha}^{i,a} &= \chi^{+,i,a} , & \Delta_\alpha \chi^{-,i,a} &= \alpha^{i,a} , \\
\Delta_\alpha \chi^{+,i,a} &= 0 , & \Delta_\alpha \alpha^{i,a} &= 0 ,
\end{aligned} \qquad (5.15)
$$

and

$$\begin{array}{ll}\Delta_\beta\overline{\beta}^{i,a}=\chi^{+,i,a}\,, & \Delta_\beta\chi^{-,i,a}=-\beta^{i,a}\,,\\ \Delta_\beta\chi^{+,i,a}=0\,, & \Delta_\beta\beta^{i,a}=0\,.\end{array} \tag{5.16}$$

Clearly, these transformations define a symmetry of the free action S_0 (5.14). However, these will also generate a symmetry of the full action (1.1). One notices that only the 3rd, 4th and 6th term of (1.2) will give rise to contributions in $\chi^\pm$. Taking a closer look at these terms, it is quite easily seen that these contributions will always be of the type

$$\chi^{+,i,a}\mathcal{O}^{ab}\chi^{-,i,b}+\overline{\beta}^{i,a}\mathcal{O}^{ab}\beta^{i,b}-\overline{\alpha}^{i,a}\mathcal{O}^{ab}\alpha^{i,b}\,, \tag{5.17}$$

with

$$\mathcal{O}^{ab}=\left\{\begin{array}{l}\delta^{ab}\\ D_\mu^{ac}D^{\mu,cb}=\delta^{ab}\partial^2-gf^{abd}\left(\partial^\mu A_\mu^d+2A_\mu^d\partial^\mu\right)+g^2f^{acd}f^{cbe}A_\mu^dA^{\mu,e}\end{array}\right. \tag{5.18}$$

and one can check that (5.17) vanishes when $\Delta_{\alpha,\beta}$ is applied to it.

It is also readily derived that

$$\{\Delta_\alpha,\Delta_\beta\}=0\,, \tag{5.19}$$

while it also holds that

$$\{\Delta_\alpha,s\}=0=\{\Delta_\beta,s\}\,, \tag{5.20}$$

since the BRST operator s acting on the new fields can be read off from (1.14), (5.7), (5.11) and (5.13) to be

$$\begin{array}{rcl}s\Omega^{a,i}&=&gf^{abc}c^b\Omega^{c,i}\,,\\ \Omega^{a,i}&\in&\left\{\alpha^{i,a},\overline{\alpha}^{i,a},\beta^{i,a},\overline{\beta}^{i,a},\chi^{+,i,a},\chi^{-,i,a},L^{i,a},N^{i,a}\right\}\,.\end{array} \tag{5.21}$$

Recapitulating, we have found 2 symmetries of the action (1.1) which are generated by the nilpotent generators (5.15) and (5.16). Moreover, the fields $(\overline{\alpha}^{i,a},\chi^{+,i,a})$ and $(\chi^{-,i,a},\alpha^{i,a})$ form Δ_α doublets, so we can be assured that these fields decouple from the physical sector by applying the **Doublet theorem**. Moreover, we can equally well deploy the **Doublet theorem** on Δ_β to also remove β and $\overline{\beta}$ from the physical subspace. The intersection $\mathrm{cohom}(\Delta_\alpha)\cap\mathrm{cohom}(\Delta_\beta)$ is thus independent of $\alpha^{i,a}$, $\overline{\alpha}^{i,a}$, $\beta^{i,a}$, $\overline{\beta}^{i,a}$, $\chi^{+,i,a}$ and $\chi^{-,i,a}$.

Consequently, we can conclude that all degrees of freedom corresponding to the extra ghost fields $G^a_{\mu\nu}$ and $\overline{G}^a_{\mu\nu}$ as well as the extra bosonic degrees of freedom corresponding to $B^a_{\mu\nu}+\overline{B}^a_{\mu\nu}$ are decoupled from the physical sector.

For the fields $N^{i,a}$ and $L^{i,a}$, corresponding to the degrees of freedom in $B^a_{\mu\nu}-\overline{B}^a_{\mu\nu}$, the analysis is less quickly performed, as these are coupled to the original gluon field A^a_μ in a nontrivial way. In the next sections, we will have a look at this problem. As we did not analyze yet the space annihilated by the free BRST charge, we may expect that certain degrees of freedom will also be killed when this subspace is considered.

6 The classical equations of motion and the Fourier decomposition of the fields

As a next step, we must quantize our model. Before turning to the conjugate momenta and quantization rules, it is advisable to have a look at the classical equations of motion in the new variables and the corresponding Fourier decompositions of their solutions. Due to the mixing between the fields, their Fourier coefficients will not all be independent.

6.1 Classical equations of motion

The free classical equation of motions are[10]

$$\begin{aligned}
b + \partial^\mu A_\mu &= 0\,, && (6.1)\\
\partial^2 A_0 - m\partial_i N^i &= 0\,, && (6.2)\\
(\partial^2 + \widetilde{m}^2)N^i - m(\partial^0 A^i - \partial^i A^0) &= 0\,, && (6.3)\\
-(\partial^2 + \widetilde{m}^2)L^i - m\varepsilon^{ijk}\partial_j A^k &= 0\,, && (6.4)\\
\partial^2 A^i + m\partial_0 N^i - m\varepsilon^{ijk}\partial_j L^k &= 0\,, && (6.5)
\end{aligned}$$

where we set

$$\widetilde{m}^2 = -\frac{3}{2}m^2\lambda_1 - \frac{1}{2}m^2\lambda_3\,. \qquad (6.6)$$

Since these equations are coupled, the quantization of the fields is not straightforward.

6.1.1 Intermezzo: multipole fields

After some manipulation with the equations (6.1)-(6.5), we derive that

$$\begin{aligned}
\partial^2\left(\partial^2 + m^2 + \widetilde{m}^2\right)(\partial^2 A_0) &= 0\,,\\
\partial^2\left(\partial^2 + m^2 + \widetilde{m}^2\right)(\partial_i N^i) &= 0\,,\\
\left(\partial^2 + \widetilde{m}^2\right)(\partial_i L^i) &= 0\,,\\
\partial^2\left(\partial^2 + m^2 + \widetilde{m}^2\right)\partial^2(\partial_i A^i) &= 0\,. && (6.7)
\end{aligned}$$

These (induced) equations of motion are of higher order in the derivatives. In the literature, such fields are known as "multipole" fields, and their quantization is indeed more involved than in the well known Klein-Gordon case. The problem of dipole fields also appears in the supergravity context [46].

Consider e.g. the following toy model of a higher derivative action [46]

$$S = -\frac{1}{2}\phi\frac{\partial^2}{m^2}\partial^2\phi\,, \qquad (6.8)$$

with classical equation of motion

$$\partial^2\partial^2\phi = 0\,. \qquad (6.9)$$

[10] We shall skip again the global color index.

It is not clear how to quantize the model (6.8) since higher order derivatives occur. We can rewrite (6.8) as

$$S = -\psi\partial^2\phi + \frac{1}{2}m^2\psi^2\,, \tag{6.10}$$

by introducing an auxiliary field ψ. It is important to notice that the derivates now occur at most quadratically, making the action (6.10) suitable for canonical quantization. In fact, this is the case we are investigating. Our action (5.12) is indeed already at most quadratical in the derivatives.

The equations of motion associated to (6.10) are

$$\begin{aligned} \partial^2\phi &= m^2\psi\,, \\ \partial^2\psi &= 0\,. \end{aligned} \tag{6.11}$$

Let us turn to Fourier space. We propose the solution

$$\psi(x) = \int \frac{d^3q}{(2\pi)^3}\widehat{\psi}(\vec{q},t)e^{i\vec{q}\vec{x}} + \text{h.c.}\,, \tag{6.12}$$

$$\phi(x) = \int \frac{d^3q}{(2\pi)^3}\widehat{\phi}(\vec{q},t)e^{i\vec{q}\vec{x}} + \text{h.c.}\,. \tag{6.13}$$

Plugging (6.12) in (6.11), the following differential equations in the time $t \equiv x_0$ arise

$$\begin{aligned} (\partial_0^2 + \vec{q}^2)\widehat{\phi}(\vec{q},t) &= m^2\widehat{\psi}(\vec{q},t)\,, \\ (\partial_0^2 + \vec{q}^2)\widehat{\psi}(\vec{q},t) &= 0\,. \end{aligned} \tag{6.14}$$

We can set

$$\widehat{\psi}(\vec{q},t) = a(\vec{q})e^{-iq_0t}\,, \qquad q_0 = |\vec{q}|\,, \tag{6.15}$$

hence the equation (6.14) is satisfied when

$$\widehat{\phi}(\vec{q},t) = \underbrace{b(\vec{q})e^{-iq_0t}}_{\text{solution hom.eq.}} + \frac{m^2}{4\vec{q}^2}(1+2i|\vec{q}|t)a(\vec{q})e^{-iq_0t}\,, \qquad q_0 = |\vec{q}|\,. \tag{6.16}$$

According to [46], terms linear in time t are typical for dipoles. As we shall soon see, terms $\propto t$ shall also appear in our massive gauge model. One can then continue by quantizing the model. The necessary commutation relations are of the type

$$\Big[a(q), b^\dagger(k)\Big] = \Big[b(q), a^\dagger(k)\Big] = \frac{1}{2k_0}\delta^{(3)}(\vec{k}-\vec{q})\,. \tag{6.17}$$

Similar, although a little more complicated techniques and relations, will occur when we try to solve the classical equations of motions of our model in Fourier space.

6.1.2 Solving the equations of motion

Let us now try to solve the equations (6.1)-(6.5) in Fourier space. We propose

$$
\begin{aligned}
A_0(x) &= \int \frac{d^3q}{(2\pi)^3} \widehat{A}_0(\vec{q},t) e^{i\vec{q}\vec{x}} + \text{h.c.}\,, \\
A^i(x) &= \int \frac{d^3q}{(2\pi)^3} \sum_{m=1}^{3} \left(\widehat{A}_m(\vec{q},t) \varepsilon^{(m),i}(\vec{q}) e^{i\vec{q}\vec{x}} \right) + \text{h.c.}\,, \\
N^i(x) &= \int \frac{d^3q}{(2\pi)^3} \sum_{m=1}^{3} \left(\widehat{N}_m(\vec{q},t) \varepsilon^{(m),i}(\vec{q}) e^{i\vec{q}\vec{x}} \right) + \text{h.c.}\,, \\
L^i(x) &= \int \frac{d^3q}{(2\pi)^3} \sum_{m=1}^{3} \left(\widehat{L}_m(\vec{q},t) \varepsilon^{(m),i}(\vec{q}) e^{i\vec{q}\vec{x}} \right) + \text{h.c.}\,, \\
b(x) &= \int \frac{d^3q}{(2\pi)^3} \left(-\partial_0 \widehat{A}_0(\vec{q},t) - i|\vec{q}|\widehat{A}_3(\vec{q},t) \right) e^{i\vec{q}\vec{x}} + \text{h.c.}\,. \qquad (6.18)
\end{aligned}
$$

The next step is to derive the equations for the Fourier coefficients. Plugging (6.18) into (6.2), we derive

$$
(\partial_0^2 + \vec{q}^2)\widehat{A}_0 = mi|\vec{q}|\widehat{N}_3\,, \qquad (6.19)
$$

while (6.3) yields

$$
\begin{aligned}
(\partial_0^2 + \vec{q}^2 + \widetilde{m}^2)\widehat{N}_3 &= m\left(\partial_0 \widehat{A}_3 + i|\vec{q}|\widehat{A}_0 \right)\,, \qquad (6.20) \\
(\partial_0^2 + \vec{q}^2 + \widetilde{m}^2)\widehat{N}_{1,2} &= m\partial_0 \widehat{A}_{1,2}\,. \qquad (6.21)
\end{aligned}
$$

(6.4) results in

$$
-(\partial_0^2 + \vec{q}^2 + \widetilde{m}^2) \sum_{m=1}^{3} \widehat{L}_m \varepsilon^{(m),i} = m\varepsilon^{ijk} i q^j \sum_{m=1}^{3} \varepsilon^{(m),k} \widehat{A}_m\,. \qquad (6.22)
$$

Multiplying with $\varepsilon^{(n),i}$, summing over i and using the orthonormality, we find

$$
-(\partial_0^2 + \vec{q}^2 + \widetilde{m}^2)\widehat{L}_n = -im \sum_{m=1}^{3} \widehat{A}_m q^j \left(\varepsilon^{(n)} \times \varepsilon^{(m)} \right)^j\,. \qquad (6.23)
$$

Taking $n = 3$, we know that $\varepsilon^{(m)} \times \varepsilon^{(n)}$ will only survive for $m = 1, 2$, however then the cross product is $\propto \varepsilon^{(2),(1)}$ and thus orthogonal to $\vec{q}$, meaning that

$$
(\partial_0^2 + q^2 + \widetilde{m}^2)\widehat{L}_3 = 0\,. \qquad (6.24)
$$

With $n = 1, 2$, (6.23) is valid if

$$
\begin{aligned}
-(\partial_0^2 + q^2 + \widetilde{m}^2)\widehat{L}_1 &= -im|\vec{q}|\widehat{A}_2\,, \qquad (6.25) \\
-(\partial_0^2 + q^2 + \widetilde{m}^2)\widehat{L}_2 &= im|\vec{q}|\widehat{A}_1\,. \qquad (6.26)
\end{aligned}
$$

Analogous manipulations on (6.5) give

$$
\begin{aligned}
(\partial_0^2+q^2)\widehat{A}_3+m\partial_0\widehat{N}_3 &= 0\,, &(6.27)\\
(\partial_0^2+q^2)\widehat{A}_1+m\partial_0\widehat{N}_1 &= -im|\vec{q}|\widehat{L}_2\,, &(6.28)\\
(\partial_0^2+q^2)\widehat{A}_2+m\partial_0\widehat{N}_2 &= im|\vec{q}|\widehat{L}_1\,. &(6.29)
\end{aligned}
$$

Let us now try to find a sensible solution to the previous differential equations in t.

We first concentrate on the independent subset of equations (6.19), (6.20) and (6.27). We can decouple these differential equations by passing to higher order differential equations. These induced equations read

$$
\begin{aligned}
(\partial_0^2+\vec{q}^2)^2(\partial_0^2+\vec{q}^2+M^2)\widehat{A}_{0,3} &= 0\,,\\
(\partial_0^2+\vec{q}^2)(\partial_0^2+\vec{q}^2+M^2)\widehat{N}_3 &= 0\,, \qquad (6.30)
\end{aligned}
$$

where we defined

$$M^2=m^2+\widetilde{m}^2\,. \qquad (6.31)$$

We solve these equations by[11]

$$
\begin{aligned}
\widehat{A}_0(\vec{q},t) &= a_0(\vec{q})e^{-i|\vec{q}|t}+t\widetilde{a}_0(\vec{q})e^{-i|\vec{q}|t}+\widehat{a}_0(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t}\,,\\
\widehat{A}_3(\vec{q},t) &= a_3(\vec{q})e^{-i|\vec{q}|t}+t\widetilde{a}_3(\vec{q})e^{-i|\vec{q}|t}+\widehat{a}_3(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t}\,,\\
\widehat{N}_3(\vec{q},t) &= n_3(\vec{q})e^{-i|\vec{q}|t}+\widehat{n}_3(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t}\,. \qquad (6.32)
\end{aligned}
$$

For consistency, we must impose the original set (6.19), (6.20) and (6.27) again, and identify the coefficients of the various t-dependent functions ($e^{-i|\vec{q}|t}$, $te^{-i|\vec{q}|t}$ and $e^{-i\sqrt{\vec{q}^2+M^2}t}$). The following equations come out as independent ones

$$
\begin{aligned}
-M^2\widehat{a}_0 &= mi|\vec{q}|\widehat{n}_3\,,\\
-2\widetilde{a}_0 &= mn_3\,,\\
\widetilde{a}_3 &= \widetilde{a}_0\,,\\
-mi|\vec{q}|a_3+m\widetilde{a}_3+mi|\vec{q}|a_0 &= \widetilde{m}^2n_3\,,\\
-M^2\widehat{a}_3 &= mi\sqrt{M^2+\vec{q}^2}\widehat{n}_3\,, \qquad (6.33)
\end{aligned}
$$

implying that there are only 3 independent coefficients left of the original 8 in (6.32).

The next step is to analyze the remaining equations. (6.24) immediately gives

$$\widehat{L}_3(\vec{q},t)=\widetilde{\ell}_3(\vec{q})e^{-i\sqrt{\vec{q}^2+\widetilde{m}^2}t}\,. \qquad (6.34)$$

The last independent set consists of (6.21),(6.25), (6.26), (6.28) & (6.29). Applying the same trick as before, we deduce

$$
\begin{aligned}
(\partial_0^2+\vec{q}^2)(\partial_0^2+\vec{q}^2+M^2)\widehat{A}_{1,2} &= 0\,,\\
(\partial_0^2+\vec{q}^2)(\partial_0^2+\vec{q}^2+\widetilde{m}^2)(\partial_0^2+\vec{q}^2+M^2)\widehat{N}_{1,2} &= 0\,,\\
(\partial_0^2+\vec{q}^2)(\partial_0^2+\vec{q}^2+\widetilde{m}^2)(\partial_0^2+\vec{q}^2+M^2)\widehat{L}_{1,2} &= 0\,, \qquad (6.35)
\end{aligned}
$$

[11] We did not write explicitly the h.c. part.

with corresponding solutions[12]

$$
\begin{aligned}
\widehat{A}_1(\vec{q},t) &= a_1(\vec{q})e^{-i|\vec{q}|t} + \widehat{a}_1(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t} \,, \\
\widehat{A}_2(\vec{q},t) &= a_2(\vec{q})e^{-i|\vec{q}|t} + \widehat{a}_2(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t} \,, \\
\widehat{L}_1(\vec{q},t) &= \ell_1(\vec{q})e^{-i|\vec{q}|t} + \widetilde{\ell}_1(\vec{q})e^{-i\sqrt{\vec{q}^2+\widetilde{m}^2}t} + \widehat{\ell}_1(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t} \,, \\
\widehat{L}_2(\vec{q},t) &= \ell_2(\vec{q})e^{-i|\vec{q}|t} + \widetilde{\ell}_2(\vec{q})e^{-i\sqrt{\vec{q}^2+\widetilde{m}^2}t} + \widehat{\ell}_2(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t} \,, \\
\widehat{N}_1(\vec{q},t) &= n_1(\vec{q})e^{-i|\vec{q}|t} + \widetilde{n}_1(\vec{q})e^{-i\sqrt{\vec{q}^2+\widetilde{m}^2}t} + \widehat{n}_1(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t} \,, \\
\widehat{N}_2(\vec{q},t) &= n_2(\vec{q})e^{-i|\vec{q}|t} + \widetilde{n}_2(\vec{q})e^{-i\sqrt{\vec{q}^2+\widetilde{m}^2}t} + \widehat{n}_2(\vec{q})e^{-i\sqrt{\vec{q}^2+M^2}t} \,.
\end{aligned}
\tag{6.36}
$$

Consistency requires

$$
\begin{array}{ll}
\widetilde{m}^2 n_1 = -im|\vec{q}|a_1 \,, & \widetilde{m}^2 n_2 = -im|\vec{q}|a_2 \,, \\
m\widehat{n}_1 = i\sqrt{\vec{q}^2+M^2}\widehat{a}_1 \,, & m\widehat{n}_2 = i\sqrt{\vec{q}^2+M^2}\widehat{a}_2 \,, \\
m\widehat{\ell}_1 = -i|\vec{q}|\widehat{a}_2 \,, & m\widehat{\ell}_2 = i|\vec{q}|\widehat{a}_1 \,, \\
n_1 = \ell_2 \,, & n_2 = -\ell_1 \,, \\
\sqrt{\vec{q}^2+\widetilde{m}^2}\widetilde{n}_1 = |\vec{q}|\widetilde{\ell}_2 \,, & \sqrt{q^2+\widetilde{m}^2}\widetilde{n}_2 = -|\vec{q}|\widetilde{\ell}_1 \,.
\end{array}
\tag{6.37}
$$

This reduces the number of independent Fourier coefficients in (6.36) from 16 to 6.

Summarizing, we have 3+1+6=10 independent Fourier coefficients left (plus their hermitian conjugates). Without loss of generality, we choose to work with $n_3(\vec{q})$, $a_1(\vec{q})$, $a_2(\vec{q})$, $\widehat{a}_1(\vec{q})$, $\widehat{a}_2(\vec{q})$, $\widehat{n}_3(\vec{q})$, $\widetilde{\ell}_3(\vec{q})$, $\widetilde{\ell}_1(\vec{q})$, $\widetilde{\ell}_2(\vec{q})$ and $a_0(\vec{q})$.

6.2 The conjugate momenta

In order to quantize the theory, we need the conjugate momenta. We shall only be concerned about the degrees of freedom hidden in the bosonic fields. We do not care about the ghosts for the moment.

Making use of the action (5.12), we derive the desired conjugate momenta,

$$
\begin{aligned}
\pi^a_i &= \frac{\delta\mathcal{L}}{\delta(\partial_0 A^{i,a})} = -F^a_{0i} - mN^a_i \,, \\
\pi^a_{N,i} &= \frac{\delta\mathcal{L}}{\delta(\partial_0 N^{i,a})} = -\partial_0 N^a_i + mA^a_i \,, \\
\pi^a_{L,i} &= \frac{\delta\mathcal{L}}{\delta(\partial_0 L^{i,a})} = \partial_0 L^a_i \,, \\
\pi^{0,a} &= b^a \equiv -\partial^\mu A^a_\mu \,.
\end{aligned}
\tag{6.38}
$$

For later use, let us compute the the multiplier b of (6.18), which is actually given by

$$
b(x) = \frac{M^2}{m}\int\frac{d^3q}{(2\pi)^3} n_3(\vec{q})e^{-i|\vec{q}|t}e^{i\vec{q}\vec{x}} + \text{h.c.} \,. \tag{6.39}
$$

[12] We assume that $\widetilde{m}^2 \neq 0$, otherwise we should adapt the analysis.

Let us now quantize the model. Naively, the brackets we would impose are[13]

$$
\begin{aligned}
[A_i(\vec{x},t),\pi_j(\vec{y},t)] &= -i\delta_{ij}\delta^{(3)}(\vec{x}-\vec{y})\,,\\
\left[A_0(\vec{x},t),\pi^0(\vec{y},t)\right] &= i\delta^{(3)}(\vec{x}-\vec{y})\,,\\
\left\{c(\vec{x},t),\pi_c(\vec{y},t)\right\} &= i\delta^{(3)}(\vec{x}-\vec{y})\,,\\
\left\{\overline{c}(\vec{x},t),\pi_{\overline{c}}(\vec{y},t)\right\} &= i\delta^{(3)}(\vec{x}-\vec{y})\,,\\
\left[N_i(\vec{x},t),\pi^N_j(\vec{y},t)\right] &= -i\delta_{ij}\delta^{(3)}(\vec{x}-\vec{y})\,,\\
\left[L_i(\vec{x},t),\pi^L_j(\vec{y},t)\right] &= i\delta_{ij}\delta^{(3)}(\vec{x}-\vec{y})\,.
\end{aligned}
\tag{6.40}
$$

However, these brackets will be not necessarily correct, due to the fact that multiple relations exist between the different field and conjugate momenta configurations, something which is clearly visible from the relations between the Fourier coefficients (6.33) and (6.37). We postpone the actual discussion of the quantization to section 7, where we shall explain an alternative way to fix the commutation relations in an appropriate fashion.

An additional complication that we should have a look at is the precise form of the free Hamiltonian $\mathcal{H}_0$. In order to have a well defined Fock state space describing physical particles, the states we are considering, which are defined by acting with the creation operators on the vacuum, should be energy eigenstates of the Hamiltonian, which should therefore be diagonalized in terms of certain creation and annihilation operators.

6.3 Calculation of the free Hamiltonian

We shall neglect that part of the Hamiltonian depending on the Faddeev-Popov ghost fields $\overline{c}^a$ and c^a, as well as depending on the fields $\alpha^{i,a}$, $\overline{\alpha}^{i,a}$, $\beta^{i,a}$, $\overline{\beta}^{i,a}$, $\chi^{+,i,a}$ and $\chi^{-,i,a}$, as these are of no relevance for the present discussion, and we call this "reduced" Hamiltonian $\mathcal{H}_0'$. Since

$$
\mathcal{H}_0' = \int d^3x\left[\pi_i\partial_0A^i + b\partial_0A_0 + \pi_{N,i}\partial_0N^i + \pi_{L,i}\partial_0L^i - \mathcal{L}\right]\,, \tag{6.41}
$$

we find

$$
\begin{aligned}
\mathcal{H}_0' &= \int d^3x\Big[-b\partial_\mu A^\mu - \frac{1}{2}b^2 + (F^{0i}+mN^i)\partial_0A^i + b\partial_0A_0 + (\partial_0N^i - mA^i)\partial_0N^i\\
&- \partial_0L^i\partial_0L^i - \frac{1}{2}F^{0i}F^{0i} + \frac{1}{4}F^{ij}F^{ij} - mN^i(\partial^0A^i - \partial^iA^0) + m\varepsilon^{ijk}L^k\partial^iA^j\\
&- \frac{1}{2}\partial_0N^i\partial_0N^i + \frac{1}{2}N^i\partial_j\partial^jN^i + \frac{1}{2}\partial_0L^i\partial_0L^i - \frac{1}{2}L^i\partial_j\partial^jL^i - \frac{1}{2}\widetilde{m}^2L^iL^i + \frac{1}{2}\widetilde{m}^2N^iN^i\Big]\,.
\end{aligned}
\tag{6.42}
$$

Since we must plug in the solutions (6.18) into the field expression of the Hamiltonian $\mathcal{H}_0'$ to find its operator valued expression, we can already use the equations of motions (6.1)-(6.5)

[13] See also the Appendix.

to simplify a bit $\mathcal{H}_0'$. Since

$$
\begin{aligned}
F^{0i} \equiv \partial^0 A^i - \partial^i A^0 &= \frac{1}{m}\left(\partial^2 + \widetilde{m}^2\right) N^i \,, \\
&\text{and} \\
\varepsilon^{ijk}\partial^i A^j &= \frac{1}{m}\left(\partial^2 + \widetilde{m}^2\right) L^k \\
\Rightarrow F^{pq} \equiv \partial^p A^q - \partial^q A^p &= \frac{1}{m}\varepsilon^{ipq}\left(\partial^2 + \widetilde{m}^2\right) L^i \\
\Rightarrow \left(\partial^p A^q - \partial^q A^p\right)^2 &= \frac{2}{m^2}\left(\partial^2 + \widetilde{m}^2\right) L^i \left(\partial^2 + \widetilde{m}^2\right) L^i \,,
\end{aligned}
\tag{6.43}
$$

we simplify $\mathcal{H}_0'$ to

$$
\begin{aligned}
& \mathcal{H}_0'\big|_{\text{on-shell}} \\
= \; & \int d^3x \left[\frac{1}{2}b^2 + \frac{1}{m}(\partial^2 + M^2)N^i\partial_0 A^i + b\partial_0 A_0 + (\partial_0 N^i - mA^i)\partial_0 N^i \right. \\
- \; & \partial_0 L^i \partial_0 L^i - \frac{1}{2m^2}(\partial^2 + \widetilde{m}^2)N^i(\partial^2 + \widetilde{m}^2)N^i + \frac{1}{2m^2}(\partial^2 + \widetilde{m}^2)L^i(\partial^2 + \widetilde{m}^2)L^i \\
- \; & N^i(\partial^2 + \widetilde{m}^2)N^i + L^i(\partial^2 + \widetilde{m}^2)L^i \\
- \; & \left. \frac{1}{2}\partial_0 N^i \partial_0 N^i + \frac{1}{2}N^i\partial_j\partial^j N^i + \frac{1}{2}\partial_0 L^i\partial_0 L^i - \frac{1}{2}L^i\partial_j\partial^j L^i - \frac{1}{2}\widetilde{m}^2 L^i L^i + \frac{1}{2}\widetilde{m}^2 N^i N^i \right] .
\end{aligned}
\tag{6.44}
$$

As a first exercise, let us have a look at the terms that give rise to the $\widetilde{\ell}_3$-oscillator. Due to (6.24), it is easy to see that the only terms relevant for this are

$$
\begin{aligned}
& \int d^3x \left[-\partial_0 L^i \partial_0 L^i + \frac{1}{2}\partial_0 L^i \partial_0 L^i - \frac{1}{2}L^i\partial_j\partial^j L^i - \frac{1}{2}\widetilde{m}^2 L^i L^i\right] \\
= \; & -2\int \frac{d^3q}{(2\pi)^3}(\vec{q}^2 + \widetilde{m}^2)\widetilde{\ell}_3^\dagger(\vec{q})\widetilde{\ell}_3(\vec{q}) \text{ after normal ordering} .
\end{aligned}
\tag{6.45}
$$

We immediately notice that we have a negative sign in the Hamiltonian as far as the $\widetilde{\ell}_3$-modes are concerned, which is inconsistent with a positive commutator for the reason of positivity (see also section 7). Invoking a negative commutator then necessarily leads to negative norm states. It might be important to notice here that the BRST charge (4.5) is of no help whatsoever to eliminate the negative norm states created by the oscillator $\widetilde{\ell}_3^\dagger$. The charge $\mathcal{Q}_0$ does only act on massless states, as implied by (4.5) and (6.39).

One might wonder if there might exist a way out, in order to find a positive Hamiltonian, without the need for negative norm states. If we could change the sign of the "new" terms in the gauge model, we should at least be able to avoid the problem in the $\widetilde{\ell}_3$ sector.

We would thus like to use the following action

$$
\begin{aligned}
S'_{phys} &= S_{cl} + S_{gf} \;, & (6.46)\\
S'_{cl} &= \int d^4x \left[-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - \frac{im}{4}(B-\overline{B})^a_{\mu\nu}F^a_{\mu\nu} - \frac{1}{4}\left(\overline{B}^a_{\mu\nu}D^{ab}_\sigma D^{bc}_\sigma B^c_{\mu\nu} - \overline{G}^a_{\mu\nu}D^{ab}_\sigma D^{bc}_\sigma G^c_{\mu\nu}\right)\right.\\
&+ \frac{3}{8}m^2\lambda_1\left(\overline{B}^a_{\mu\nu}B^a_{\mu\nu} - \overline{G}^a_{\mu\nu}G^a_{\mu\nu}\right) - m^2\frac{\lambda_3}{32}\left(\overline{B}^a_{\mu\nu} - B^a_{\mu\nu}\right)^2\\
&+ \left.\frac{\lambda^{abcd}}{16}\left(\overline{B}^a_{\mu\nu}B^b_{\mu\nu} - \overline{G}^a_{\mu\nu}G^b_{\mu\nu}\right)\left(\overline{B}^c_{\rho\sigma}B^d_{\rho\sigma} - \overline{G}^c_{\rho\sigma}G^d_{\rho\sigma}\right)\right] \;, & (6.47)
\end{aligned}
$$

which is completely similar to the action (1.1) up to a few signs.

The steps in the algebraic renormalizability analysis of [1, 2] are not essentially affected by these sign changes. More precisely, it is still possible to show the renormalizability to all orders. It is clear that the supersymmetry (5.6) is still present for $m = 0$, so that the equivalence with ordinary Yang-Mills theories is maintained for $m \equiv 0$.

In the decomposed form, we have the following (quadratic) action in Minkowski space time

$$
\begin{aligned}
S_0 &= \int d^4x \left[-\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 - mN^{i,a}\left(\partial^0 A^{ia} - \partial^i A^{0a}\right) + m\varepsilon^{ijk}L^{k,a}\left(\partial^i A^{ja}\right)\right.\\
&- \frac{1}{2}\chi^{+,i,a}\partial^2\chi^{-,i,a} + \frac{1}{2}N^{i,a}\partial^2 N^{i,a} - \frac{1}{2}L^{i,a}\partial^2 L^{i,a} - \frac{1}{2}\overline{\beta}^{i,a}\partial^2\beta^{i,a} + \frac{1}{2}\overline{\alpha}^{i,a}\partial^2\alpha^{i,a}\\
&+ \frac{3}{4}m^2\lambda_1\left(\chi^{+,i,a}\chi^{-,i,a} - N^{i,a}N^{i,a} + L^{i,a}L^{i,a} + \overline{\beta}^{i,a}\beta^{i,a} - \overline{\alpha}^{i,a}\alpha^{i,a}\right)\\
&- \left. m^2\frac{\lambda_3}{4}\left(N^{i,a}N^{i,a} - L^{i,a}L^{i,a}\right) + b^a\partial_\mu A^{\mu a} + \overline{c}^a\partial^2 c^a + \frac{\xi}{2}b^a b^a\right] \;. & (6.48)
\end{aligned}
$$

Evidently, the cohomology analysis of section 4 can be immediately translated into the new language.

The classical equations of motion read

$$
\begin{aligned}
b + \partial^\mu A_\mu &= 0\,, & (6.49)\\
\partial^2 A_0 + m\partial_i N^i &= 0\,, & (6.50)\\
(\partial^2 + \widetilde{m}^2)N^i - m(\partial^0 A^i - \partial^i A^0) &= 0\,, & (6.51)\\
-(\partial^2 + \widetilde{m}^2)L^i - m\varepsilon^{ijk}\partial_j A^k &= 0\,, & (6.52)\\
\partial^2 A^i - m\partial_0 N^i + m\varepsilon^{ijk}\partial_j L^k &= 0\,. & (6.53)
\end{aligned}
$$

We solve them by using once more the Fourier decomposition (6.18), yielding the following differential equations in time t

$$
\begin{aligned}
(\partial_0^2 + \vec{q}^2)\widehat{A}_0 &= -mi|\vec{q}|\widehat{N}_3\,,\\
(\partial_0^2 + \vec{q}^2 + \widetilde{m}^2)\widehat{N}_3 &= m\left(\partial_0\widehat{A}_3 + i|\vec{q}|\widehat{A}_0\right)\,,\\
(\partial_0^2 + q^2)\widehat{A}_3 - m\partial_0\widehat{N}_3 &= 0\,, & (6.54)
\end{aligned}
$$

and

$$
\begin{aligned}
(\partial_0^2+\vec{q}^2+\widetilde{m}^2)\widehat{N}_{1,2} &= m\partial_0\widehat{A}_{1,2}\,,\\
(\partial_0^2+q^2+\widetilde{m}^2)\widehat{L}_3 &= 0\,,\\
(\partial_0^2+q^2+\widetilde{m}^2)\widehat{L}_1 &= im|\vec{q}|\widehat{A}_2\,,\\
(\partial_0^2+q^2+\widetilde{m}^2)\widehat{L}_2 &= -im|\vec{q}|\widehat{A}_1\,,\\
(\partial_0^2+q^2)\widehat{A}_1-m\partial_0\widehat{N}_1 &= im|\vec{q}|\widehat{L}_2\,,\\
(\partial_0^2+q^2)\widehat{A}_2-m\partial_0\widehat{N}_2 &= -im|\vec{q}|\widehat{L}_1\,.
\end{aligned}
\tag{6.55}
$$

Decoupling the first set of equations (6.54) leads to

$$
\begin{aligned}
(\partial_0^2+\vec{q}^2)^2(\partial_0^2+\vec{q}^2+M^2)\widehat{A}_{0,3} &= 0\,,\\
(\partial_0^2+\vec{q}^2)(\partial_0^2+\vec{q}^2+M^2)\widehat{N}_3 &= 0\,,
\end{aligned}
\tag{6.56}
$$

where the mass M^2 is now defined by

$$
M^2=\widetilde{m}^2-m^2\,. \tag{6.57}
$$

The presence of the mass scale $\widetilde{m}^2\neq 0$ is essential here to avoid the appearance of a tachyonic mass $-m^2$, which is due to the change of sign in the action (6.46). The mass scales $\propto\lambda_i m^2$ leading to the mass $\widetilde{m}^2$ were originally introduced in [1] to ensure that the final action is renormalizable from the algebraic point of view. In the present context, these also play a physical role in order to avoid tachyons, if we assume that $\widetilde{m}^2>m^2$.

The solution to (6.56) is still given by (6.32), however the relations between the coefficients are modified into

$$
\begin{aligned}
M^2\widehat{a}_0 &= mi|\vec{q}|\widehat{n}_3\,,\\
2\widetilde{a}_0 &= mn_3\,,\\
\widetilde{a}_3 &= \widetilde{a}_0\,,\\
-mi|\vec{q}|a_3+m\widetilde{a}_3+mi|\vec{q}|a_0 &= \widetilde{m}^2n_3\,,\\
M^2\widehat{a}_3 &= mi\sqrt{M^2+\vec{q}^2}\widehat{n}_3\,.
\end{aligned}
\tag{6.58}
$$

It is straightforward to verify that

$$
b(x) = \int\frac{d^3q}{(2\pi)^3}\left(\frac{M^2}{m}\right)n_3(\vec{q})e^{-i|\vec{q}|t}e^{i\vec{q}\vec{x}}+\text{h.c.}\,. \tag{6.59}
$$

The conjugate momenta are given by

$$
\begin{aligned}
\pi_i^a &= \frac{\delta\mathcal{L}}{\delta(\partial_0A^{i,a})}=-F_{0i}^a+mN_i^a\,,\\
\pi_{N,i}^a &= \frac{\delta\mathcal{L}}{\delta(\partial_0N^{i,a})}=\partial_0N_i^a-mA_i^a\,,\\
\pi_{L,i}^a &= \frac{\delta\mathcal{L}}{\delta(\partial_0L^{i,a})}=-\partial_0L_i^a\,,\\
\pi^{0,a} &= b^a\equiv-\partial^\mu A_\mu^a\,.
\end{aligned}
\tag{6.60}
$$

The equation of motion for $\widehat{L}_3$ is immediately solved by (6.34).

Let us now turn to the $(1,2)$-sector. It is not difficult to check that (6.35) and (6.36) still holds, but now with

$$\begin{array}{ll} \widetilde{m}^2 n_1 = -im|\vec{q}|a_1\,, & \widetilde{m}^2 n_2 = -im|\vec{q}|a_2\,, \\ m\widehat{n}_1 = -i\sqrt{\vec{q}^2+M^2}\widehat{a}_1\,, & m\widehat{n}_2 = -i\sqrt{\vec{q}^2+M^2}\widehat{a}_2\,, \\ m\widehat{\ell}_1 = i|\vec{q}|\widehat{a}_2\,, & m\widehat{\ell}_2 = -i|\vec{q}|\widehat{a}_1\,, \\ n_1 = \ell_2\,, & n_2 = -\ell_1\,, \\ \sqrt{\vec{q}^2+\widetilde{m}^2}\widetilde{n}_1 = |\vec{q}|\widetilde{\ell}_2\,, & \sqrt{q^2+\widetilde{m}^2}\widetilde{n}_2 = -|\vec{q}|\widetilde{\ell}_1\,. \end{array} \tag{6.61}$$

The (reduced) free Hamiltonian $\mathcal{H}_0'$ becomes

$$\begin{aligned} \mathcal{H}_0' &= \int d^3x \left[-b\partial_\mu A^\mu - \frac{1}{2}b^2 + (F^{0i} - mN^i)\partial_0 A^i + b\partial_0 A_0 + mA^i\partial_0 N^i \right. \\ &- \frac{1}{2}F^{0i}F^{0i} + \frac{1}{4}F^{ij}F^{ij} + mN^i(\partial^0 A^i - \partial^i A^0) - m\varepsilon^{ijk}L^k\partial^i A^j \\ &- \left. \frac{1}{2}\partial_0 N^i\partial_0 N^i - \frac{1}{2}N^i\partial_j\partial^j N^i + \frac{1}{2}\partial_0 L^i\partial_0 L^i + \frac{1}{2}L^i\partial_j\partial^j L^i + \frac{1}{2}\widetilde{m}^2 L^i L^i - \frac{1}{2}\widetilde{m}^2 N^i N^i \right]. \end{aligned} \tag{6.62}$$

Since the relations (6.43) are still valid, we find for the first part

$$\begin{aligned} & \mathcal{H}_0'\big|_{\text{on-shell}} \\ &= \int d^3x \left[\frac{1}{2}b^2 + \frac{1}{m}(\partial^2 + M^2)N^i\partial_0 A^i + b\partial_0 A_0 + mA^i\partial_0 N^i \right. \\ &- \frac{1}{2m^2}(\partial^2+\widetilde{m}^2)N^i(\partial^2+\widetilde{m}^2)N^i + \frac{1}{2m^2}(\partial^2+\widetilde{m}^2)L^i(\partial^2+\widetilde{m}^2)L^i \\ &+ N^i(\partial^2+\widetilde{m}^2)N^i - L^i(\partial^2+\widetilde{m}^2)L^i - \frac{1}{2}\partial_0 N^i\partial_0 N^i - \frac{1}{2}N^i\partial_j\partial^j N^i \\ &+ \left. \frac{1}{2}\partial_0 L^i\partial_0 L^i + \frac{1}{2}L^i\partial_j\partial^j L^i + \frac{1}{2}\widetilde{m}^2 L^i L^i - \frac{1}{2}\widetilde{m}^2 N^i N^i \right]. \end{aligned} \tag{6.63}$$

After some tedious algebra, we can express the part of the Hamiltonian corresponding to the (massive) $\sim$-modes as

$$\begin{aligned} \mathcal{H}_0'^{,(1)} &= \int \frac{d^3q}{(2\pi)^3} \left[2\left(\vec{q}^2+\widetilde{m}^2\right)\widetilde{\ell}_3^\dagger(\vec{q})\widetilde{\ell}_3(\vec{q}) + 2\left(\vec{q}^2+\widetilde{m}^2\right)\widetilde{\ell}_1^\dagger(\vec{q})\widetilde{\ell}_1(\vec{q}) + 2\left(\vec{q}^2+\widetilde{m}^2\right)\widetilde{\ell}_2^\dagger(\vec{q})\widetilde{\ell}_2(\vec{q}) \right. \\ &- \left. 2\left(\vec{q}^2+\widetilde{m}^2\right)\widetilde{n}_1^\dagger(\vec{q})\widetilde{n}_1(\vec{q}) - 2\left(\vec{q}^2+\widetilde{m}^2\right)\widetilde{n}_2^\dagger(\vec{q})\widetilde{n}_2(\vec{q}) \right]. \end{aligned} \tag{6.64}$$

Using the relations (6.58) and (6.61), we rewrite (6.64) as

$$\mathcal{H}_0'^{,(1)} = \int \frac{d^3q}{(2\pi)^3} \left[2\left(\vec{q}^2+\widetilde{m}^2\right)\widetilde{\ell}_3^\dagger(\vec{q})\widetilde{\ell}_3(\vec{q}) + 2\widetilde{m}^2\widetilde{\ell}_1^\dagger(\vec{q})\widetilde{\ell}_1(\vec{q}) + 2\widetilde{m}^2\widetilde{\ell}_2^\dagger(\vec{q})\widetilde{\ell}_2(\vec{q}) \right]. \tag{6.65}$$

The second part, corresponding the (massive) $\wedge$-modes is given by

$$\mathcal{H}_0'^{,(2)} = \int \frac{d^3q}{(2\pi)^3} \left[-2(\vec{q}^2+M^2)\left(1+\frac{m^2}{M^2}\right)\widehat{n}_3^\dagger(\vec{q})\widehat{n}_3(\vec{q}) - 2\widetilde{m}^2\widehat{n}_1^\dagger(\vec{q})\widehat{n}_1(\vec{q}) - 2\widetilde{m}^2\widehat{n}_2^\dagger(\vec{q})\widehat{n}_2(\vec{q}) \right]. \tag{6.66}$$

Finally, we also have a part $\mathcal{H}_0'^{,(3)}$ corresponding to the massless modes, which we do not write in more detail.

7 Fixing the commutation relations between creation and annihilation operators and the non-unitarity of the model

7.1 Outline of the idea

We recall here that the equations of motion (6.49)-(6.53) constitute several (nontrivial) relations between the field components. Unfortunately, these relations are quite complicated to solve explicitly. Closely related to this is the fact that it is not immediately clear how to diagonalize the quadratic form appearing in the action (6.48). Moreover, this would-be diagonalized form should also be of maximum second order in the derivatives to allow for a consistent quantization.

Since the fields are related to each other, we can expect the same for the conjugate momenta. E.g., it is easily checked that $\pi_0(\vec{x},t) = -\pi_3(\vec{x},t)$.

It would appear that quantization, starting from configuration space, is a highly nontrivial task due to the existing relationships between the fields and momenta. It would be beneficial to quantize the theory directly in Fourier (momentum) space, in which case the independent degrees of freedom are clearly identifiable.

There does exist a simple way to derive the appropriate (anti-)commutation relations between the creation and annihilation operators, when we rely on the Heisenberg picture. We can always calculate the free Hamiltonian operator $\mathcal{H}_0$ without exact knowledge of the commutation relations. The precise value of the commutators only influences the constant part of $\mathcal{H}_0$, which we can drop after normal ordering. On one hand, we already know the time evolution of all fields Ψ, as written down in (6.18) by using the Fourier decomposition and the solutions (6.32) and (6.36). On the other hand, these fields are treated as operators in the Heisenberg picture, and consequently their time evolution is dictated by the Heisenberg equation

$$[\mathcal{H}_0, \Psi] = -i\frac{\partial}{\partial t}\Psi\,. \tag{7.1}$$

Demanding consistency allows to fix the commutators between the creation and annihilation operators, which are supposed to be number valued. As an illustration, consider the Klein-Gordon action

$$S_{KG} = \int d^4x \left(-\frac{1}{2}\varphi\left(\partial^2 + m^2\right)\varphi\right)\,. \tag{7.2}$$

The solution to the equation of motion is expressed by

$$\begin{aligned}\varphi(\vec{x},t) &= \int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q}a(\vec{q})e^{-i\omega_q t}e^{i\vec{q}\vec{x}} + \text{h.c.}\\ \omega_q &= \sqrt{\vec{q}^2+m^2}\,.\end{aligned} \tag{7.3}$$

As it is well known, the normal ordered Hamiltonian is given by[14]

$$\mathcal{H}_{KG} = \int \frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_q}\omega_q a^\dagger(\vec{q})a(\vec{q})\,, \tag{7.4}$$

[14]We disregard the conventional $:\mathcal{H}_{KG}:$ notation and continue to use $\mathcal{H}_{KG}$.

so that requiring

$$[\mathcal{H}_{KG}, \varphi(\vec{x},t)] = -i\frac{\partial}{\partial t}\varphi(\vec{x},t)\,, \tag{7.5}$$

leads to the correct commutation relation

$$\left[a(\vec{q}), a^\dagger(\vec{k})\right] = 2\omega_q (2\pi)^3 \delta^{(3)}(\vec{q}-\vec{k})\,. \tag{7.6}$$

7.2 Quantization of the massive gauge model using the Hamiltonian $\mathcal{H}_0'$

In order to find the commutators for the massive $\sim$-modes, we only need to consider $\mathcal{H}_0^{\prime,(1)}$. When we impose

$$\left[\mathcal{H}_0^{\prime,(1)}, \widehat{L}^i(\vec{q},t)\right] = -i\frac{\partial}{\partial t}\widehat{L}^i(\vec{q},t)\,, \tag{7.7}$$

we deduce that the following commutation relations arise

$$\begin{aligned}
\left[\widehat{\ell}_{1,2}(\vec{q}), \widehat{\ell}^\dagger_{1,2}(\vec{k})\right] &= (2\pi)^3 \frac{\sqrt{\vec{q}^2+\widetilde{m}^2}}{2\widetilde{m}^2}\delta^{(3)}(\vec{k}-\vec{q})\,,\\
\left[\widehat{\ell}_3(\vec{q}), \widehat{\ell}^\dagger_3(\vec{k})\right] &= (2\pi)^3 \frac{1}{2\sqrt{\vec{q}^2+\widetilde{m}^2}}\delta^{(3)}(\vec{k}-\vec{q})\,.\\
\text{others trivial} &
\end{aligned} \tag{7.8}$$

We can define novel operators as follows

$$\begin{aligned}
\widetilde{\ell}_{1,2}(\vec{q}) &= \frac{1}{2\widetilde{m}}\widetilde{\ell}'_{1,2}(\vec{q})\,,\\
\widetilde{\ell}_3(\vec{q}) &= \frac{1}{2\sqrt{\vec{q}^2+\widetilde{m}^2}}\widetilde{\ell}'_3(\vec{q})\,,
\end{aligned} \tag{7.9}$$

to arrive at a free Hamiltonian in a more standard form

$$\mathcal{H}_0^{\prime,(1)} = \int \frac{d^3q}{(2\pi)^3}\frac{1}{2\widetilde{\omega}_q}\left[2\widetilde{\omega}_q \sum_{i=1}^{3} \widetilde{\ell}'^\dagger_i(\vec{q})\widetilde{\ell}'_i(\vec{q})\right]\,, \tag{7.10}$$

with commutation relations

$$\begin{aligned}
\left[\widetilde{\ell}'_i(\vec{q}), \widetilde{\ell}'^\dagger_j(\vec{k})\right] &= \widetilde{\omega}_q(2\pi)^3\delta_{ij}\delta^{(3)}(\vec{k}-\vec{q})\,,\\
\text{others trivial}\,. &
\end{aligned} \tag{7.11}$$

The energy was defined as

$$\widetilde{\omega}_q = \sqrt{\vec{q}^2+\widetilde{m}^2}\,. \tag{7.12}$$

We conclude that the modes $\widetilde{\ell}'_i(\vec{q})$ correspond to the 3 polarizations of a vector particle with mass $\widetilde{m}$. As it is clear from (7.11), they have a positive norm.

If we would have used the original Hamiltonian (6.44), we would have found a negative sign commutator, as already advocated below (6.63).

In a completely similar fashion, we can quantize the $\wedge$-modes. We rewrite (6.66) into the form

$$\mathcal{H}_0^{\prime,(2)} = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\widehat{\omega}_q} \left[-\widehat{\omega}_q \sum_{i=1}^{3} \widehat{n}_i^{\prime\dagger}(\vec{q}) \widehat{n}_i^{\prime}(\vec{q}) \right] , \tag{7.13}$$

by a suitable rescaling of the fields, in which case the corresponding commutation relations become

$$\begin{aligned} \left[\widehat{n}_i^{\prime}(\vec{q}), \widehat{n}_j^{\prime\dagger}(\vec{k}) \right] &= -2\widehat{\omega}_q (2\pi)^3 \delta_{ij} \delta^{(3)}(\vec{k} - \vec{q}) \\ \text{others trivial} . \end{aligned} \tag{7.14}$$

with

$$\widehat{\omega}_q = \sqrt{\vec{q}^2 + M^2} . \tag{7.15}$$

We conclude that we have been able to change the negative norms corresponding to the $\widetilde{\ell}_i$-modes in our first model (1.1) by passing to (6.47), but unfortunately the negative norm problem has been "shifted" to the $\widehat{n}_i$-modes, as it is clearly visible from (7.14). We are thus forced to conclude that our theory is not unitary, as we do not seem to be able to remove these negative norm states, corresponding to the polarizations of a massive vector particle with mass M. Just as in the case of the $\widetilde{\ell}_i$-modes, we reckon once more that the BRST symmetry has nothing to say about the $\widehat{n}_i$-modes.

To completely finish the analysis, we should in principle also determine the free Hamiltonian corresponding to the massless modes. However, this has become a bit redundant job by now since we already have established that the theory cannot be unitary. Quite obviously, some restriction on the allowed (massless) states and Faddeev-Popov ghost states will arise from the BRST charge (4.5). However, this might also turn out to be a not so trivial task after all due to the multipole character, encoded in (6.32) in the terms linear in time t. As it was found in the case of the toy model of [46], the Hamiltonian is not even necessarily diagonal, obscuring the particle interpretation. It is even noticed that the Hamiltonian cannot be diagonalized with linear transformations. As already said, we shall however not dwell on that point here.

8 Conclusion

We continued our investigation of a recently proposed local non-Abelian gauge invariant action containing mass terms (1.1), which can be renormalized to all orders of perturbation once a gauge is chosen. Despite the fact that the model enjoys a BRST symmetry with nilpotent generator given in (1.14), it turned out to be impossible to remove all negative norm states from the physical state space. As a consequence, our model is not unitary and thus not useful as a physical theory containing massive particles as asymtotic observables.

As already indicated in the introduction, this does not mean that our action is now useless. We can still wonder whether it could be generated dynamically in a nonperturbative fashion when we would start with the initially massless version of our gauge model. As a matter

of fact, we can propose our model as an alternative to ordinary Yang-Mills theory in the perturbative (massless) region, since both are equivalent [2]. It might however be more convenient to search for a dynamical mass m in our model since the mass term can be at least coupled to the action without spoiling the renormalizability or gauge invariance. Of course, it remains to be investigated if a sensible gap equation could be established. If this would work out, we might have a gauge invariant mechanism behind the dynamical generation of a massive parameter into a gauge theory.

We repeat that the generation of a mass would not be necessarily in conflict with the nonunitarity of the massive gauge model, since we start from the (unitary) massless theory and we do no longer want to describe the asymptotic high energy behaviour wherefore perturbation theory applies perfectly, but rather we are entering a phenomenologically interesting region where e.g. the gluons already loose their physical meaning as an observable.

Since our generated mass would be gauge invariant, it could enter physical correlation functions. As such, it could be investigated if it could serve as a possible alternative to the $\langle A^2\rangle_{\min}$ condensate used to explain the $\frac{1}{Q^2}$ power corrections [6, 7].

We conclude by noticing that we could repeat our analysis in the Abelian case. Then it can be shown that the action (1.1) is stable without the need for λ_1 and λ_3 [1]. It can also be shown that the model is equivalent with the Abelian Stueckelberg model [47], described by

$$\begin{aligned} S_{\text{stuck}} &= -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}\int d^4x (A_\mu + \partial_\mu\phi)(A^\mu + \partial^\mu\phi) \\ &+ \int d^4x \left(b\partial A + \frac{1}{2}b^2 + \overline{c}\partial^2 c \right) . \end{aligned} \tag{8.1}$$

When the auxiliary fields are integrated out in both cases, some manipulation leads to the same (nonlocal) action

$$S_{\text{nonlocal}} = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{4}\int d^4x F_{\mu\nu}\frac{1}{\partial^2}F^{\mu\nu} + \int d^4x \left(b\partial A + \frac{1}{2}b^2 + \overline{c}\partial^2 c \right) \tag{8.2}$$

It is known that the Abelian Stueckelberg model is renormalizable and unitary, see e.g. [47] for a review. If we would analyze the unitarity of the Abelian version of (1.2), we would run into exactly the same problem as in the non-Abelian case, i.e. the presence of negative norm states in the physical subspace.

The lesson to be learnt is the following. We depart from the same nonlocal action, but in order to give a consistent quantization of the theory, including an analysis of the unitarity and renormalizability, we are forced to bring the action in some localized polynomial form. Apparently, the exact procedure of localization affects these results. For example, in the Abelian case, there are 2 quite distinct approaches to bring the action (8.2) in a local and in addition renormalizable form: the Stueckelberg approach or the pathway we followed. However, only the Stueckelberg way gives a unitary model. Apparently, the precise role of the additional fields that are introduced cannot be underestimated in the discussion of the unitarity and/or renormalizability.

Acknowledgments

D. Dudal would like to thank S. P. Sorella for useful discussions. D. Dudal is a postdoctoral fellow of the *Special Research Fund* of Ghent University

Appendix

Since we have been working with the decomposed fields, the covariance of the (naive) commutation relations (6.40) is obscured. It is worth having a look at this. We decompose $B_{\mu\nu}$ and its complex conjugate $\overline{B}_{\mu\nu}$ into their real and imaginary part.

$$X_{\mu\nu} = \frac{1}{2}(B_{\mu\nu} + \overline{B}_{\mu\nu}) \qquad Y_{\mu\nu} = \frac{1}{2i}(B_{\mu\nu} - \overline{B}_{\mu\nu}) \,, \tag{8.3}$$

then

$$\begin{array}{ll} M_i = X_{0i} \,, & N_i = Y_{0i} \,, \\ K^i = \frac{1}{2}\varepsilon^{ijk} X_{jk} \,, & L^i = \frac{1}{2}\varepsilon^{ijk} Y_{jk} \,. \end{array} \tag{8.4}$$

Covariance and the antisymmetry would require commutation relations like

$$\begin{aligned} \left[X_{\mu\nu}(\vec{x},t), \pi^X_{\alpha\beta}(\vec{y},t)\right] &= i\left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right)\delta^{(3)}(\vec{x}-\vec{y}) \,, \\ \left[Y_{\mu\nu}(\vec{x},t), \pi^Y_{\alpha\beta}(\vec{y},t)\right] &= i\left(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}\right)\delta^{(3)}(\vec{x}-\vec{y}) \,. \end{aligned} \tag{8.5}$$

Specifically

$$\begin{aligned} \left[X_{0i}(\vec{x},t), \pi^X_{0j}(\vec{y},t)\right] &= i g_{ij}\delta^{(3)}(\vec{x}-\vec{y}) = -i\delta_{ij}\delta^{(3)}(\vec{x}-\vec{y}) \,, \\ \left[X_{ij}(\vec{x},t), \pi^X_{kl}(\vec{y},t)\right] &= i\left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right)\delta^{(3)}(\vec{x}-\vec{y}) \,, \end{aligned} \tag{8.6}$$

and the same for Y. Consequently

$$\begin{aligned} \left[M_i(\vec{x},t), \pi^M_j(\vec{y},t)\right] &= -i\delta_{ij}\delta^{(3)}(\vec{x}-\vec{y}) \,, \\ \left[K_r(\vec{x},t), \pi^K_s(\vec{y},t)\right] &= \frac{1}{4}\varepsilon^{rij}\varepsilon^{skl}\left[X_{ij}(\vec{x},t), \pi^X_{kl}(\vec{y},t)\right] \\ &= i\frac{1}{4}\varepsilon^{rij}\varepsilon^{skl}\left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right)\delta^{(3)}(\vec{x}-\vec{y}) = i\delta_{rs}\delta^{(3)}(\vec{x}-\vec{y}) \,, \end{aligned} \tag{8.7}$$

and the same for N and L.

References

[1] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Verschelde, *A study of the gauge invariant, nonlocal mass operator $Tr\int d^4x F_{\mu\nu}\frac{1}{D^2}F_{\mu\nu}$ in Yang-Mills theories*, Phys. Rev. D **72** (2005) 105016.

[2] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella and H. Verschelde, *Quantum properties of a non-Abelian gauge invariant action with a mass parameter*, Phys. Rev. D **74** (2006) 045008.

[3] T. van Ritbergen, A. N. Schellekens and J. A. M. Vermaseren, *Group theory factors for Feynman diagrams*, Int. J. Mod. Phys. A **14** (1999) 41.

[4] J. M. Cornwall, *Dynamical Mass Generation In Continuum QCD*, Phys. Rev. D **26** (1982) 1453.

[5] K. G. Chetyrkin, S. Narison and V. I. Zakharov, *Short-distance tachyonic gluon mass and $\frac{1}{Q^2}$ corrections*, Nucl. Phys. B **550** (1999) 353.

[6] F. V. Gubarev, L. Stodolsky and V. I. Zakharov, *On the significance of the vector potential squared*, Phys. Rev. Lett. **86** (2001) 2220.

[7] F. V. Gubarev and V. I. Zakharov, *On the emerging phenomenology of $\langle A_\mu^2 \rangle$*, Phys. Lett. B **501** (2001) 28.

[8] V. N. Gribov, *Quantization Of Non-Abelian Gauge Theories*, Nucl. Phys. B **139** (1978) 1.

[9] Semenov-Tyan-Shanskii and V.A. Franke, Zapiski Nauchnykh Seminarov Leningradskogo Otdeleniya Matematicheskogo Instituta im. V.A. Steklov AN SSSR, Vol. **120** (1982) 159. English translation: New York: Plenum Press 1986.

[10] M. Lavelle and D. McMullan, *Constituent quarks from QCD*, Phys. Rept. **279** (1997) 1.

[11] P. Boucaud, A. Le Yaouanc, J. P. Leroy, J. Micheli, O. Pene and J. Rodriguez-Quintero, *Testing Landau gauge OPE on the lattice with a A^2 condensate*, Phys. Rev. D **63** (2001) 114003.

[12] M. J. Lavelle and M. Schaden, *Propagators And Condensates In QCD*, Phys. Lett. B **208** (1988) 297.

[13] K. I. Kondo, *Vacuum condensate of mass dimension 2 as the origin of mass gap and quark confinement*, Phys. Lett. B **514** (2001) 335.

[14] H. Verschelde, K. Knecht, K. Van Acoleyen and M. Vanderkelen, *The non-perturbative groundstate of QCD and the local composite operator A_μ^2*, Phys. Lett. B **516** (2001) 307.

[15] D. Dudal, H. Verschelde, R. E. Browne and J. A. Gracey, *A determination of $\langle A_\mu^2 \rangle$ and the non-perturbative vacuum energy of Yang-Mills theory in the Landau gauge*, Phys. Lett. B **562** (2003) 87.

[16] D. Dudal, H. Verschelde and S. P. Sorella, *The anomalous dimension of the composite operator A^2 in the Landau gauge*, Phys. Lett. B **555** (2003) 126.

[17] G. Parisi and R. Petronzio, *On Low-Energy Tests Of QCD*, Phys. Lett. B **94** (1980) 51.

[18] F. Halzen, G. I. Krein and A. A. Natale, *Relating the QCD pomeron to an effective gluon mass*, Phys. Rev. D **47** (1993) 295.

[19] J. H. Field, *A phenomenological analysis of gluon mass effects in inclusive radiative decays of the J/psi and Upsilon*, Phys. Rev. D **66** (2002) 013013.

[20] P. Marenzoni, G. Martinelli and N. Stella, *The Gluon propagator on a large volume, at beta = 6.0*, Nucl. Phys. B **455** (1995) 339.

[21] K. Langfeld, H. Reinhardt and J. Gattnar, *Gluon propagators and quark confinement*, Nucl. Phys. B **621** (2002) 131.

[22] K. Amemiya and H. Suganuma, *Effective mass generation of off-diagonal gluons as the origin of infrared Abelian dominance in the maximally Abelian gauge in QCD*, Phys. Rev. D **60** (1999) 114509.

[23] V. G. Bornyakov, M. N. Chernodub, F. V. Gubarev, S. M. Morozov and M. I. Polikarpov, *Abelian dominance and gluon propagators in the maximally Abelian gauge of SU(2) lattice gauge theory*, Phys. Lett. B **559** (2003) 214.

[24] A. C. Aguilar and A. A. Natale, *A dynamical gluon mass solution in a coupled system of the Schwinger-Dyson equations*, JHEP **0408** (2004) 057.

[25] A. C. Aguilar and J. Papavassiliou, *Gluon mass generation in the PT-BFM scheme*, JHEP **0612** (2006) 012.

[26] D. Dudal, H. Verschelde, V. E. R. Lemes, M. S. Sarandy, S. P. Sorella and M. Picariello, *Gluon-ghost condensate of mass dimension 2 in the Curci-Ferrari gauge*, Annals Phys. **308** (2003) 62.

[27] D. Dudal, H. Verschelde, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella, M. Picariello and J. A. Gracey, *The anomalous dimension of the gluon-ghost mass operator in Yang-Mills theory*, Phys. Lett. B **569** (2003) 57.

[28] D. Dudal, H. Verschelde, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella and J. A. Gracey, *Renormalizability of the local composite operator A_μ^2 in linear covariant gauges*, Phys. Lett. B **574** (2003) 325.

[29] R. E. Browne and J. A. Gracey, *Two loop effective potential for $\langle A_\mu^2 \rangle$ in the Landau gauge in quantum chromodynamics*, JHEP **0311** (2003) 029.

[30] D. Dudal, H. Verschelde, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro and S. P. Sorella, *Dynamical gluon mass generation from $\langle A_\mu^2 \rangle$ in linear covariant gauges*, JHEP **0401** (2004) 044.

[31] D. Dudal, J. A. Gracey, V. E. R. Lemes, M. S. Sarandy, R. F. Sobreiro, S. P. Sorella and H. Verschelde, *An analytic study of the off-diagonal mass generation for Yang-Mills theories in the maximal Abelian gauge*, Phys. Rev. D **70** (2004) 114038.

[32] R. E. Browne and J. A. Gracey, *One loop MS-bar gluon pole mass from the LCO formalism*, Phys. Lett. B **597** (2004) 368

[33] J. A. Gracey, *Two loop MS-bar gluon pole mass from the LCO formalism*, Eur. Phys. J. C **39** (2005) 61.

[34] V. E. R. Lemes, R. F. Sobreiro and S. P. Sorella, *Renormalizability of the dimension two gluon operator A^2 in a class of nonlinear covariant gauges*, J. Phys. A **40** (2007) 4025.

[35] D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella, R. Thibes and H. Verschelde, *Remarks on a class of renormalizable interpolating gauges*, JHEP **0507** (2005) 059.

[36] R. Jackiw and S. Y. Pi, *Threshhold Singularities and the Magnetic Mass in Hot QCD*, Phys. Lett. B **368** (1996) 131.

[37] C. Becchi, A. Rouet and R. Stora, *Renormalization Of Gauge Theories*, Annals Phys. **98** (1976) 287.

[38] I. V. Tyutin, Lebedev Institute preprint (unpublished), 1975.

[39] T. Kugo and I. Ojima, *Local Covariant Operator Formalism Of Nonabelian Gauge Theories And Quark Confinement Problem*, Prog. Theor. Phys. Suppl. **66** (1979) 1.

[40] A. A. Slavnov, *Physical Unitarity In The Brst Approach*, Phys. Lett. B **217** (1989) 91.

[41] S. A. Frolov and A. A. Slavnov, *Construction Of The Effective Action For General Gauge Theories Via Unitarity*, Nucl. Phys. B **347** (1990) 333.

[42] M. E. Peskin and D. V. Schroeder, *An Introduction To Quantum Field Theory*, Addison-Wesley Publishing Company (1995).

[43] H. Lehmann, K. Symanzik and W. Zimmermann, *On the formulation of quantized field theories*, Nuovo Cim. **1** (1955) 205.

[44] M. Henneaux and C. Teitelboim, *Quantization of gauge systems*, Princeton University Press (1992).

[45] O. Piguet and S. P. Sorella, *Algebraic renormalization: Perturbative renormalization, symmetries and anomalies*, Lect. Notes Phys. **M28** (1995) 1.

[46] S. Ferrara and B. Zumino, *Structure of linearized supergravity and conformal supergravity* , Nucl. Phys. B **134** (1978) 301.

[47] H. Ruegg and M. Ruiz-Altaba, *The Stueckelberg field*, Int. J. Mod. Phys. A **19** (2004) 3265.